Contracting Institutions and Economic Growth

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Abstract

This paper studies the effects of contracting institutions on economic development. A growth model is presented with endogenous incomplete markets, where financial frictions generated by the imperfect enforcement of contracts depend on the difference between future and current output. The former determines the costs of being excluded from financial markets after defaulting. The latter determines the benefits of not servicing current obligations. As the economy approaches its balanced growth path, frictions and their effect on income become more important because the net benefits of honoring contracts decrease as this difference narrows. Therefore, as the economy approaches its steady-state, the effect of contracting institutions on GDP per capita increases. This result is proven analytically and its robustness is explored in a more general quantitative model of heterogeneous agents and incomplete markets. The model delivers a testable prediction which is confirmed using simulated data, and validated empirically using a modification of previous specifications of cross-country regressions on institutions and per capita GDP.

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1 Introduction

A central question in economics is how to explain the large and persistent differences we observe in per capita income across countries. One view relates these differences to the organization of society, or its institutions. An extensive empirical literature, described in La Porta et al. (2008), investigates the link between legal institutions and income per capita, finding a strong and significant relationship. These types of institutions are not only related to the rules governing contracting among private agents, they are also a determinant of a broader set of rules related to the protection of property rights (Levine, 2005). Acemoglu and Johnson (2005) distinguish between two different types of institutions: contracting institutions (CI) - those that enable private contracts between citizens - and property rights institutions (PRI) - those that protect citizens against expropriation by the government and powerful elites. They point out a particular feature of CI that would be behind their finding that these cannot explain empirically differences in income per capita across countries: in the case of weak CI, the terms of contracts can be modified to protect citizens from opportunistic behavior, something that is not possible in the case of weak PRI due to the impossibility of constraining those who control the state.

This particular feature of private contracts appears in the literature on finance and development through collateral constraints. Since lenders can appropriate defaulted borrowers' assets, contracts are contingent on these. Hence the intensity of financial frictions and the effects of CI depend on the financial wealth of individuals. However this literature has not explored other types of punishments to defaulters, such as future exclusion from markets. This is key in the literature on endogenous incomplete markets which, on the other hand, hasn't explored issues relating to long-run development.[1] In this paper we explore analytically the implications of this type of punishment in a simple growth model. We find, in the absence of any change in the nature of CI, a double causality relationship between economic development and financial contracting. Frictions affecting contracting reduce income per capita, as in most of the literature. The novel finding of the paper is the reverse causality: the closer the economy to its balanced growth path, the lower the self-enforcement incentives, and hence the larger the effects of CI on income per capita since they are more likely to determine the set of contracts that are feasible in equilibrium.

Therefore, the main contribution of the paper deals with the effects of institutions on transitional dynamics, and as such, the paper relates to a growing literature that studies the impact of financial frictions on the latter, i.e. Marcet and Marimon (1992), Giné and Townsend (2004), Jeong and Townsend (2007), Song et al. (2011) Buera and Shin (2013), Moll (2014). As this literature illustrates, focusing on transitions allows to better analyze the consequences of reforms and observed growth dynamics, which differ from the fast convergence implied by the neoclassical growth model. In our environment the effects of a growth-enhancing reform may have a larger effect in the presence of inefficient CI, since it may render them irrelevant. But as diminishing returns slow down the effects of the reform on growth, these inefficient institutions may become binding, affecting income per capita again.[2].

In the model, financial frictions arise from the assumption that entrepreneurs, who borrow resources in order to invest in physical capital, cannot commit to honor their contacts. Hence, the penalties associated with default become an important component of contracting. It is assumed

[1] The exception is Marcet and Marimon (1992). See the literature review section below for further details.

[2] Results also highlight the different effects of institutions at different levels of development, as in Acemoglu et al. (2006) and, more recently, Gancia and Bonfiglioli (2013)

that one of these penalties concerns the ability of entrepreneurs who have defaulted to take full advantage of future production opportunities, a consequence of the exclusion from financial markets that restricts the amount of resources available to be invested in the future. But default allows appropriation of the resources borrowed from consumers, which are proportional to current output in equilibrium. Thus, the net benefit of honoring the contract is increasing in the expected future growth of the economy, as a higher growth rate makes future production opportunities, or future borrowing, more attractive relative to default. Therefore, financial frictions become less binding in quickly-growing economies and more efficient contracts are self-enforced. Because the literature on finance and development has not considered exclusion from future production opportunities as a punishment, this is a novel mechanism.

Furthermore, the benefit of defaulting is decreasing in CI quality, which is assumed to be exogenous in the model. Thus, even if self-enforcement incentives are weak, optimal contracts can be enforced if the institutional quality is good enough. But if the efficient contract is self-enforced in the absence of these institutions, due to high expected growth, the quality of the latter does not affect production.

The paper embeds these financial frictions into the standard neoclassical growth model. Along the transition path towards the steady-state growth is declining and thus self-enforcement weakens, reaching its lowest level in the steady-state. Since the level of output does not have an independent effect on self-enforcement incentives besides its influence on the growth rate, this result is independent from other distortions affecting the steady-state level of income per capita. Therefore financial frictions are more important when the economy is close to its balanced growth path. The main prediction of the model is that the effect of the quality of CI on income per capita across countries becomes significant only after some fraction of the steady-state capital stock has been accumulated. Only after this happens do debt constraints generated by financial frictions become binding.

In order to introduce a need for external borrowing in the model in a simplified way, entrepreneurs are assumed to be unable to save. This allows us to prove analytically the main result regarding the timing of the effect of CI on output. A more realistic case where entrepreneurs can save may influence the main mechanism in the model. In this case assets could be used as collateral and hence the cost of defaulting would include, besides foregone future production opportunities, asset lost. The net gains from defaulting in this case would depend not only on future growth but also on the ratio of entrepreneurs' assets to current output. If this ratio grows fast enough during the transition, self-investment could offset the main mechanism of the model.

Is it the case that entrepreneurs' assets grow much faster than output? One reason to observe this in equilibrium is that, since external financing is increasing in entrepreneurs' wealth, a faster accumulation alleviates financial constraints.[3] But notice that the mechanism in the model relates to the timing at which these constraints become more likely binding. Therefore, if expected growth is high and default incentives are low, financial frictions are less relevant and hence asset accumulation is closer to the frictionless case. As growth slows down during the transition to the balanced growth path, incentives to default raise and so does asset accumulation. Distortions are increasing since entrepreneurs move away from their efficient consumption path, and they are maximized in steady-state. Thus, if assets are not enough to make entrepreneurs unconstrained at that stage, increasing output costs should be observed as well. The result would be that CI are still relatively

[3] It is worth noting however that the transition to the steady state need not coincide with the life cycle of entrepreneurs, as more realistically agents transit in and out of entrepreneurship. Also note that self investment relates to the absolute level of output, not to its level relative to its steady-state level as the mechanism in the model.

more influent the closer the economy is to its balanced growth path.

To explore if this intuition is correct we extend the simple model to allow asset accumulation by entrepreneurs with concave preferences. In this new environment, agents face idiosyncratic shocks and along the balanced growth path financial frictions generate output costs. We solve the model numerically and show that these costs decrease with the distance of the economy to its steady-state. Results show also that the fraction of agents that are financially constrained rises along the convergence to the steady-state, and the same happens with the minimum level of collateral, relative to the level of capital, allowing them to produce using the efficient level of factors of production.

We perform a simple empirical exercise to validate the main mechanism of the model, minimally modifying the identification strategy and econometric specifications of Acemoglu and Johnson (2005), who find no significant effects of CI on income per capita after controlling for PRI, to introduce the notion of conditional convergence (Barro, 1991; Barro and Sala-i-Martin, 1992, 2004).[4] In this case, as regressions with simulated data show, because we control for other determinants of the steady-state level of output per capita, the coefficient of CI should be increasing in the initial level of GDP per capita since the higher is initial GDP, the lower is expected growth. This testable prediction is supported by the econometric exercise, which also confirms the results of Acemoglu and Johnson (2005). We find that the effect of CI on growth in output per capita in the last 60 years was significant only for countries that were relatively close to their steady states in 1950.

After a brief literature review, the next section presents the model. It first characterizes the competitive equilibrium under perfect enforcement of contracts, and then it describes the imperfect enforcement equilibrium. Section 3 presents the numerical extension of the model to allow for entrepreneurs' self-investment. Section 4 presents the evidence, and the last section concludes.

1.1 Literature Review

This paper is closely related to the theoretical literature on financial frictions and growth.[5] Since exclusion from future production opportunities as a punishment has not been introduced in these papers, the main implication of the model presented is a new contribution. The exception would be Marcet and Marimon (1992). Although they do not explicitly refer to the effect of decreasing growth rates on the participation constraint during the transition to the steady state, the mechanism should be present in their environment. But their results are different from the ones presented here in that they analyze the central planner problem, allowing transfers between lenders and borrowers contingent on default decisions. Then, the fact that growth is decreasing over time implies a path for borrowers' consumption that is increasing over time.[6] Moreover, the optimal level of investment is feasible in steady state as contingent transfers to borrowers are positive.

Since the literature on financial frictions and growth, focusing on imperfect enforcement, has ignored the dynamics incentives that exclusion from financial markets introduce, collateral is the

[4] It is not the aim of the paper to falsify the results of Acemoglu and Johnson (2005). Indeed we confirm them in the empirical part. What the model does in this dimension is to define certain conditions under which finding a strong effect of CI on income per capita for a cross-section of countries may be difficult, even if the distortions generated by suboptimal contracts are important.

[5] Most of it focuses on informational imperfections instead of imperfect enforcement. For instance Townsend (1979), Greenwood and Jovanovic (1990), Castro et al. (2004, 2009) Townsend and Ueda (2006), and Greenwood et al. (2010).

[6] This is also due to the presence of stochastic productivity shocks, which lead to increasing payments over time, as in Holmström (1983), Thomas and Worrall (1988), and Kocherlakota (1996).

main influence on default decisions. We introduce collateral in Section 3 in a quantitative framework. An important issue in this dimension is whether entrepreneurs are able to overcome financial frictions entirely, at least in the long run, in this new environment. The closest to this paper and most informative in this respect is Moll (2014), who studies analytically this problem not only in a stationary equilibrium but also during transitions. The main finding is that, since it takes time to accumulate enough assets, the existence –and, more specifically, the persistence of productivity shocks– influence the effectiveness of self-financing in overcoming financial frictions.[7]

Other papers study this feature numerically. For example, Jeong and Townsend (2007), Amaral and Quintin (2010), Buera et al. (2011), Buera and Shin (2013), and Midrigan and Xu (2014) present quantitative models of limited enforcement and development where savings decisions are relevant but do not overcome financial frictions in the long run. Productivity shocks, limited life spans, and nonconvexities are behind these results.[8] Based on these findings we build the general model such that frictions do affect output in steady state. Then we explore whether distortions are increasing during the transition. Note that collateral constraints bind when agents are poor, while in the mechanism we propose constraints bind late in the development process. This issue is discussed in detail in Section 3.

This paper extends the basic framework of the relationship between finance and development. It introduces dynamic incentives in defaulting choices in a model of growth. Thereby it can be situated at the intersection of the literature on finance and development and the one studying endogenous incomplete markets theoretically, particularly the implications of introducing exclusion from financial markets after defaulting, although not in a growth context. Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000) study limited enforceability of contracts and imperfect insurance, while Eaton and Gersovitz (1981), Cole and Kehoe (1995), Kletzer and Wright (2000), and Kehoe and Perri (2002) study sovereign borrowing where enforcement by a third party is totally absent. Among the papers on imperfect insurance and incomplete markets, the closest to this paper is the one by Krueger and Perri (2006), who also study the effect of changes in the environment on self-enforcement incentives, although in a different context.

Empirically this paper is related to the extensive literature exploring the link between institutions and income per capita. Papers focusing on the role of legal institutions have found a close link between their quality and the origin of legal systems. Levine (2005) argues that legal systems that embrace jurisprudence, such as British common law, tend to adapt more efficiently to the changing contractual needs of an economy than legal systems that adhere rigidly to formalistic procedures and coded law, such as French civil law countries. Beck et al. (2003) present evidence which is consistent with this channel. Some of the outcomes influenced by the origin of legal systems are investor protection (La Porta et al., 1997, 1998), the formalism of judicial procedures (Djankov et al., 2003), judicial independence (La Porta et al., 2004), and the quality of contract enforcement (Djankov et al., 2008). Using these findings some papers have identified a strong and significant relationship between these institutions and income per capita (Beck et al., 2000; Levine, 1998, 1999; Levine et al., 2000). As noted above, Acemoglu and Johnson (2005) explore the comparative effects

[7]Unlike most of the literature, Moll (2014) also analyzes transitions to steady states analytically as we do. However, his focus is different. He analyzes how the environment (i.e. the nature of idiosyncratic shocks) affects the size of the costs generated by financial frictions. We analyze, given a shock structure, not the size but the timing of these cost during a transition, which may be triggered by changes in the environment.

[8]In the theoretical work by Albuquerque and Hopenhayn (2004) self-investment make entrepreneurs unconstrained in the long run. In their environment borrowers have linear preferences, no dividends are distributed, and all profits are paid to the lender until the entrepreneur becomes unconstrained.

of CI and PRI on income per capita. They document a strong link between CI and legal origin on the one hand, and PRI and initial endowments on the other, which would have influenced the type of institutions established by Europeans in former colonies.

2 The Model

2.1 The Economic Environment

The economy is populated by workers ($i = w$) and entrepreneurs ($i = e$), each with measure 1. There is no entry to, or exit from, entrepreneurship.[9] Entrepreneurs have access to the following technology to produce output,

$$y_t = z_t k_t^{\alpha} n_t^{\upsilon} \tag{1}$$

where z captures the level of technology, k and n are capital and labor used in production, respectively, and α and υ are positive constants. There are decreasing returns to scale, so $\omega = 1-\alpha-\upsilon > 0$. Finally z grows at constant (gross) rate $\mu > 1$, so this is a deterministic growth model. The representative type i agent maximizes the expected value of his lifetime utility as given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i) = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^i)^{1-\sigma^i} - 1}{1-\sigma^i}$$

where σ^i is the risk aversion coefficient or the inverse of the elasticity of substitution. It is assumed for simplicity that entrepreneurs are risk neutral, so $\sigma^i = 0$ for $i = e$. This assumption is relaxed in the next section. Workers are risk averse so $\sigma^w = \sigma > 0$. Capital depreciates at rate δ each period, implying the following market clearing condition,

$$C_t = \sum_i c_t^i = Y_t + (1-\delta)K_t - K_{t+1} \tag{2}$$

where c^i is total consumption by agents $i = w, e$, and capital letters denote aggregate variables.

Workers save an amount b out of their income and lend it to entrepreneurs, who do not save. Entrepreneurs finance capital with these resources and, if they find it optimal to do so, they repay workers the amount lent plus the market interest rate after production takes place. Hence we only consider one period contracts, which are not constrained optimal in this environment. Entrepreneurs can default. Lenders can always tell whether an entrepreneur has defaulted, but courts can verify this and force the repayment only with probability $(1-\rho)$.[10] Thus the parameter ρ captures the quality of institutions related to the enforcement of contracts. If entrepreneurs find it optimal not to repay workers and if they are not caught defaulting, they appropriate the stock of capital and its return. In case of default, entrepreneurs cannot borrow further, but they can

[9] This simplifying assumption together with decreasing returns to scale implies the existence of positive profits in the long run. But since lending is constrained at an individual level due to the possibility of default, entry could overcome the effects of financial frictions. However, if entrepreneurs differ in their productivity, new entrants will be less productive and therefore a misallocation of entrepreneurial ability will reduce output as well.

[10] The monitoring technology becomes a critical issue when introducing exclusion as punishment in low-income countries, as lenders generally lack the ability to monitor borrowers. However, there is extensive literature showing that the lack of information available for screening borrowers reinforces local credit relationships (Cull et al., 2006; Fafchamps, 2004; Kumar and Matsusaka, 2009), making alternative credit relationships more difficult to establish, and validating exclusion as a punishment.

use the installed capital to produce in the future with the same technology described above. This assumption, that entrepreneurs keep the capital stock after defaulting and do not save otherwise, is relaxed later in the quantitative model.[11] If the entrepreneur is caught, which happens with probability $(1-\rho)$, he is forced to give back the capital stolen plus the return, and he is also excluded from financial markets, leaving him without any income source for the future, an assumption which is also relaxed in the quantitative model.

2.2 Recursive Competitive Equilibrium

The aggregate state of the world is described by K, which evolution is governed by the function $K' = \mathrm{K}(K)$, which is exogenously given for all agents. The dynamic program problem facing the representative entrepreneur is

$$V(K) = \max_{k,n} \left\{c + \beta V(K')\right\}$$

$$st. \quad c = y - w(K)n - (r(K) + \delta)k,$$

and an incentive compatibility (IC) constraint,[12]

$$c + \beta V(K') \geq \rho \left[y - w(K,z)n + \beta V^d((1-\delta)k; K')\right] + (1-\rho)c.$$

This IC constraint ensures that the entrepreneur does not find it optimal to default, and reflects the fact that the market anticipates default decisions.[13] The function $V^d(\bar{k}'; K')$ is the continuation value of defaulting, and it is defined by the following problem,

$$V^d(\bar{k}'; K') = \max_{n'} \left\{\bar{y}' - w'(K')n' + \beta V^d(\bar{k}''; K'')\right\}$$

$$\begin{aligned} st. \quad \bar{y}' &= z'\bar{k}'^{\alpha} n'^{\upsilon} \\ \bar{k}'' &= (1-\delta)\bar{k}' \end{aligned}$$

The dynamic problem facing the representative worker is standard, as he only observes prices,

$$U(b; K) = \max_{c,b'} \left\{u(c) + \beta U(b'; K')\right\}$$

$$st. \quad c + b' = w(K) + b(1 + r(K))$$

The worker is also constrained by the standard transversality condition. The competitive equilibrium can now be defined.

Definition 1. *A competitive equilibrium is a set of decision functions $c^w = \mathrm{C}(K)$, $b' = \mathrm{B}(K)$, and $n = \mathrm{N}(K)$, a set of pricing functions $w = \mathrm{W}(K)$ and $r = \mathrm{R}(K)$, and an aggregate law of motion for the capital stock $K' = \mathrm{K}(K)$, such that,*

[11] Regardless, what happens to the entrepreneur after defaulting is not critical for the main result, since this is not due to a relatively low growth of consumption after defaulting. It is shown below that the result holds even if this rate is the same with and without access to financial markets, and if the punishment is temporary or permanent. The important feature is that it is not possible to achieve, in every period after defaulting, the level of capital that would be invested with access to financial markets.

[12] The assumption that defaulters consume $c > 0$ with probability $1-\rho$ helps with the algebra later.

[13] This constraint may be imposed by financial intermediaries, from which entrepreneurs may borrow only until they do not find it optimal to default.

1. *Entrepreneurs solve their dynamic programming problem, given* $\mathrm{K}(\cdot)$, $\mathrm{W}(\cdot)$ *and* $\mathrm{R}(\cdot)$, *with the equilibrium solution satisfying* $n = \mathrm{N}(k)$.

2. *Workers solve their dynamic programming problem, given* $\mathrm{K}(\cdot)$, $\mathrm{W}(\cdot)$ *and* $\mathrm{R}(\cdot)$, *with the equilibrium solution satisfying* $c^w = \mathrm{C}(K)$ *and* $b' = \mathrm{B}(K)$.

3. *Market clearing conditions,* $C = Y + (1-\delta)K - K'$ *and* $N = 1$, *hold each period.*

It is easy to demonstrate that this model converges to a balanced growth path.[14] Therefore given the conjectured asymptotic growth rate for all variables, which we denote by γ, one can impose a transformation that will render them stationary in the limit. This transformation consists of defining the new variables $\hat{x}_t = x_t/g_x^t$, where g_x is the growth rate of some variable x_t when $t \to \infty$. The transformed dynamic programming problems are presented in Appendix A. The main differences with respect to the original model are the discount factors, which now incorporate all information related to the non transitional dynamics of the economy. Now $\beta\gamma$ is the discount factor for workers and entrepreneurs that have not defaulted, while for entrepreneurs that have previously defaulted the discount rate is now $\beta\bar{\gamma} = \beta\gamma^{\omega/(1-\upsilon)}(1-\delta)^{\alpha/(1-\upsilon)} < \beta\gamma$.

2.3 Perfect Enforceability (PE)

When there is perfect enforcement of contracts, i.e. $\rho = 0$, the model is simplified to the standard neoclassical growth model, but with decreasing returns to scale. In this case, entrepreneurs equalize marginal productivities to factor prices and workers set consumption growth according to a standard Euler equation. A well known result for this kind of model, to be proved below, is that $\forall \hat{K} < \hat{K}^{\mathrm{ss}}$, $\Delta\hat{K} > 0$, where $\hat{K}^{\mathrm{ss}}$ is the transformed level of capital in steady state and Δ denotes the one-period change in a variable. As capital increases in the transition to the steady state, output and wages rise while the interest rate falls. This implies that during the transition the interest rate is higher than the subjective discount rate, generating an increasing path for consumption. An additional feature of the PE equilibrium, which is key in analyzing the IE equilibrium later, is that the rate of growth (or decrease) of all variables falls during the transition. As the return on capital falls when the economy approaches its steady state, capital accumulation slows down, lowering output growth, the growth rate of wages, and the rate at which the interest rate decreases. The next proposition describes the transition of the economy from an initial low capital stock to its balanced growth path under PE.

Proposition 1. *Suppose* $\rho = 0$ *and* $\hat{K} < \hat{K}^{ss}$. *Then,*

$$\Delta\hat{K} > 0,\ \Delta\hat{w} > 0,\ \Delta r < 0,\ \Delta\hat{C} > 0,\ \text{and } \Delta\hat{Y} > 0,$$

and,

$$\Delta\left|\frac{\Delta x}{x}\right| = \Delta\left|g_x\right| < 0,$$

for $x = \hat{K}, \hat{w}, r, \hat{C}, \hat{Y}$.

Proof. See Appendix B. □

[14] In particular, along this path the interest rate is constant and c^w grows at a constant rate. Let us call this rate γ. Since all the variables on the RHS of equation (2) and c^e, grow at a constant rate, they must do so at the same rate γ. Moreover, using the production function, we have that $\gamma = \mu\gamma^{\alpha}$, which implies $\gamma = \mu^{1/(1-\alpha)}$. Notice that this is true whether the IC constraint is binding or not. The asymptotic growth rates off the equilibrium path are derived in Appendix A.

2.4 Imperfect Enforceability (IE)

Under IE of contracts, i.e. $\rho > 0$, the IC constraint is relevant, so we now study its binding pattern. First rewrite the IC constraint of the transformed problem described in Appendix A as follows,

$$IC(\hat{K}) = \beta \left[\gamma V(\hat{K}') - \rho\bar{\gamma}V^d(\hat{k};\hat{K}')\right] - \rho\left[\hat{y} - \hat{w}(\hat{K}) - \hat{c}\right]$$

Here the first term of the RHS is the future cost of defaulting, while the second term is the current benefit of defaulting. In equilibrium, $IC(\hat{K}) \geq 0$. Since entrepreneurs are risk neutral, the current benefit of defaulting is just $\rho(\hat{y} - \hat{w}(\hat{K}) - \hat{c}) = \rho(r(\hat{K}) + \delta)\hat{k}$. Thus, in equilibrium, the following must hold:

$$IC(\hat{K}) = \beta \left[\gamma V(\hat{K}') - \rho\bar{\gamma}V^d(\hat{k};\hat{K}')\right] - \rho(r(\hat{K}) + \delta)\hat{k} \geq 0. \tag{3}$$

Using the optimal demand for labor, the continuation value of honoring the contract, $V(\hat{K}')$, and of defaulting, $V^d(\hat{k};\hat{K}')$, can be expressed recursively by

$$V(\hat{K}') = (1-\upsilon)\hat{y}' - (r' + \delta)\hat{k}' + \beta\gamma V(\hat{K}'')$$

$$V^d(\hat{k};\hat{K}') = (1-\upsilon)\hat{z}\hat{k}^\alpha n'^\upsilon + \beta\bar{\gamma}V^d(\hat{k};\hat{K}'')$$

So the next period's flow utility if the entrepreneur honors the contract is output net of factor payments, while utility if the entrepreneur defaults is the value of output, using the stock of capital acquired at the moment of defaulting (which is constant in the transformed problem), net of labor income.

First let us analyze the steady state of this economy. In this case all endogenous variables are constant, so condition (3) holds if and only if

$$IC(\hat{K}^{ss}) = \phi\left(1 + \frac{\omega}{\alpha}\right)\ \alpha\frac{\hat{y}^{ss}}{\hat{k}^{ss}} - (r^{ss} + \delta) \geq 0$$

where

$$\phi = \left(\frac{\beta\gamma}{1-\beta\gamma} - \frac{\beta\rho\bar{\gamma}}{1-\beta\bar{\gamma}}\right) / \left(\rho + \frac{\beta\gamma}{1-\beta\gamma}\right) > 0.$$

It is easy to see that if $\phi(1 + \omega/\alpha) \geq 1$, the constraint will not bind in steady state and the PE allocation, where $\alpha\hat{y}^{ss}/\hat{k}^{ss} = r^{ss} + \delta$, will result. Otherwise imperfect enforceability of contracts distorts the steady-state equilibrium. The following proposition formalizes this result and establishes the existence and uniqueness of the equilibrium.

Proposition 2. *There is a unique steady-state equilibrium with a locally unique path leading to it. In particular, the following holds in steady state,*

$$\Omega\ \alpha\frac{\hat{Y}^{ss}}{\hat{K}^{ss}} = \frac{\gamma^\sigma}{\beta} - 1 + \delta = r^{ss} + \delta,$$

where

$$\Omega = min\left\{1, \phi\left(1 + \frac{\omega}{\alpha}\right)\right\}.$$

Proof. See Appendix B. □

As in the standard neoclassical growth model, there is a unique steady state. Using a linear approximation in the neighborhood of the steady state, the proposition also shows that there is a locally unique path leading to it.[15] The proposition also shows that, in steady state, the IC constraint will be more likely to bind in a sector which is more capital intensive –the larger is α– and in the sector with lower rents under first best allocations –the lower is ω. Finally, since ϕ is decreasing in ρ, the constraint will be tighter and, if binding, the distortion will be larger, the larger is this parameter. Also notice that if $\rho = 0$, then $\phi = 1$, and the constraint is not binding in steady state. If the constraint is binding in steady state, then a lower level of capital must be observed in equilibrium, so the entrepreneur finds it optimal to honor the contract and repay the workers. This raises the output to capital ratio relative to the PE case, in line with the condition in Proposition 2. As output falls when the constraint binds, labor demand and wages also fall.

During the transition the incentives to default depend on the future path of the economy. Any effect of the constraint in the future will change equilibrium allocations, affecting how binding the current constraint is as well. However, it is useful to first analyze the constraint assuming that PE allocations hold throughout the transition and in steady state. In order to do this, use the FOC for capital to replace $(r+\delta)\hat{k}$ with $\alpha\hat{y}$ in expression (3). Rearranging terms and aggregating over all the entrepreneurs, we can express the IC constraint under PE allocations (IC^{PE}) as

$$IC^{\mathrm{PE}}(\hat{K}) = \beta\left[\gamma\tilde{V}(\hat{K},\hat{K}') - \rho\bar{\gamma}\tilde{V}^d(\hat{K},\hat{K}')\right] - \rho\alpha \geq 0 \tag{4}$$

where

$$\begin{aligned}
\tilde{V}(\hat{K},\hat{K}') &= (1-\alpha-\upsilon)\left[\frac{\hat{Y}'}{\hat{Y}} + \beta\gamma\frac{\hat{Y}''}{\hat{Y}} + (\beta\gamma)^2\frac{\hat{Y}'''}{\hat{Y}} + ...\right] \\
&= (1-\alpha-\upsilon)\,\frac{\hat{Y}'}{\hat{Y}} + \beta\gamma\tilde{V}(\hat{K},\hat{K}'') \\
\tilde{V}^d(\hat{K},\hat{K}') &= (1-\upsilon)\left[\left(\frac{\hat{w}}{\hat{w}'}\right)^{\frac{\upsilon}{1-\upsilon}} + \beta\bar{\gamma}\left(\frac{\hat{w}}{\hat{w}''}\right)^{\frac{\upsilon}{1-\upsilon}} + (\beta\bar{\gamma})^2\left(\frac{\hat{w}}{\hat{w}'''}\right)^{\frac{\upsilon}{1-\upsilon}} + ...\right] \\
&= (1-\upsilon)\left(\frac{\hat{w}}{\hat{w}'}\right)^{\frac{\upsilon}{1-\upsilon}} + \beta\bar{\gamma}\tilde{V}^d(\hat{K},\hat{K}'').
\end{aligned}$$

We can interpret $\tilde{V}$ and $\tilde{V}^d$ as the continuation utility of honoring and not honoring contracts relative to the current gain. Then, under PE allocations, the relative continuation utility of honoring the contract depends positively on the future growth rate of output. The higher is the former, the higher are rents when the entrepreneur maintains access to consumer savings relative to the amount which she is able to steal today. Likewise, the relative continuation utility of defaulting depends negatively on the future growth on wages. If the entrepreneur is excluded from financial markets then the future path for rents is totally determined by the cost of the only variable factor of

[15] To analyze the global dynamics of the economy, the transversality condition and similar arguments used for the standard model can be applied here. A higher growth rate in the stock of capital during the transition may exist if agents expect higher growth rates in the future. But this can only be sustained if capital grows forever, which violates the transversality condition. On the other hand, lower growth rates would make the capital stock hit zero in finite time, violating the consumer's maximization problem.

production, labor.[16] As described above, under PE allocations the growth rate of wages and output slows down as time passes. Therefore, the relative continuation utility of honoring the contract, $\tilde{V}$, decreases over time, while the relative continuation utility of not honoring the contract, $\tilde{V}^d$, increases over time. Notice that what appears in the denominator (numerator) inside the square brackets in the expression for $\tilde{V}$ ($\tilde{V}^d$) is current output (wages). This shows that what matters for the default decision is future output relative to current output, which is realized before default, and not the growth rate after this happens. Hence the assumptions regarding the growth rate of consumption for entrepreneurs after defaulting is not behind the main result of the model, being possible to take a more general approach for modelling the outside option.[17] This discussion leads to the following proposition.

Proposition 3. *If $\Omega = 1$, PE allocations are the outcome $\forall t$ if $\hat{K}_0 \leq \hat{K}^{ss}$. Otherwise, if $\Omega < 1$, $\forall \rho \in [0,1]$, $\exists K^* > 0$, where $K^* \leq \hat{K}^{ss}$, such that, if $\hat{K} < K^*$, $IC(\hat{K}) > 0$, and if $K^* \leq \hat{K}$, $IC(\hat{K}) = 0$.*

Proof. See Appendix B. □

As the economy approaches its steady state, defaulting becomes more attractive, as the net cost of doing so decreases. Since the cost converges monotonically to its steady-state value, this implies that the IC constraint will not be binding at any point during the transition if it is not binding along the balanced growth path. Otherwise it must bind at some point. The proposition shows that this happens at some positive level of capital for any value of ρ, meaning that there is always some range for capital where the constraint is not binding. This result derives from the fact that the marginal productivity of capital, and so the growth rate of output, converges to infinity as capital converges to zero.

Additionally, proposition 3 states that once the constraint is binding, it never ceases to bind. Above we showed that under PE allocations the value of honoring the contract is decreasing during the transition. But the statement here is stronger since it takes into account future IE allocations. The intuition is similar, though. At any level of capital during the transition, tomorrow's capital is lower under a binding constraint than under a non-binding one. Hence if there was already a point where growth under a non-binding constraint was insufficient to support the efficient contract, and since growth under a non-binding constraint is decreasing, growth under a binding constraint will never be enough to support the efficient contract again.

[16] Mathematically we divide V^d by $\hat{y}$ and the first term of $\tilde{V}^d$ becomes $(1-\upsilon)(n'/n)^{\upsilon}$. The entrepreneurs' optimality condition implies $n = \upsilon\hat{y}/\hat{w}$, but this is not true for n' because capital after defaulting is not at the optimal level given prices. The condition is $n' = (\upsilon\hat{y}/n^{\upsilon}\hat{w}')^{1/(1-\upsilon)}$. Replacing them above we get the expression for $\tilde{V}^d$.

[17] To clarify this point consider for a moment a case in which defaulters have access to some technology so as to get the same consumption growth rate as non-defaulters. In this case we would have

$$\tilde{V}^d(\hat{K}, \hat{K}') = (1-\upsilon)\frac{\kappa\hat{Y}'}{\hat{Y}} + \beta\gamma\tilde{V}^d(\hat{K}, \hat{K}'') = \frac{\kappa(1-\upsilon)}{1-\alpha-\upsilon}\tilde{V}(\hat{K}, \hat{K}')$$

where $\kappa \in (0,1)$ affects the level but not the growth rate of consumption. Then

$$IC^{\text{PE}}(\hat{K}) = \beta\gamma\left(1 - \frac{\rho\kappa(1-\upsilon)}{1-\alpha-\upsilon}\right)\tilde{V}(\hat{K}, \hat{K}') - \rho\alpha \geq 0$$

which, as long as the entrepreneur cannot take full advantage of future production opportunities (i.e. the term inside the parenthesis is positive), $IC^{\text{PE}}(\hat{K})$ becomes more likely to bind as $\tilde{V}$ decreases along the transition.

The IE equilibrium can now be characterized. If $\Omega \leq 1$, at some point PE allocations are no longer feasible and capital accumulation slows down because of the fall in the interest rate. Thus $r\hat{k}$ falls in expression (3), so the constraint holds with equality. From proposition 2 we know that, as the economy converges to the steady state, a larger fraction of the adjustment is achieved through the lower stock of capital since the interest rate needs to converge to its exogenous steady-state value. This adjustment is unambiguously bad for workers. Before the constraint binds, as future expected income falls, they increase savings and so the aggregate stock of capital during that period is larger than in the PE equilibrium outcome. The following proposition compares the path for capital under PE and IE.

Proposition 4. *Take the sequence* $\{\hat{\boldsymbol{K}}_s\}_{s=0}^{\infty}$ *as the equilibrium sequence of capital under PE* ($\rho = 0$)*. Fix* $\hat{K}_0 = \hat{\boldsymbol{K}}_0$*. Then, if* $\Omega < 1$, $\exists K^{**} > K^{*}$*, such that, if* $\hat{K}_t = K^{**}$, $\hat{K}_s > \hat{\boldsymbol{K}}_s$ *if* $0 < s < t$*, and* $\hat{K}_s < \hat{\boldsymbol{K}}_s$ *if* $s > t$*.*

Proof. See Appendix B. □

The left panel of Figure 1 shows the path for capital under PE and under IE when the constraint is binding in steady state. Below the first threshold for capital, $\hat{K}^{*}$ as defined in proposition 3, the constraint is not binding, but higher savings generate a faster growth of capital under IE. Above this point the difference in capital levels closes as the constraint becomes binding and so the growth rate of capital falls under IE relative to PE allocations. The gap is eliminated when the economy achieves the level of capital $\hat{K}^{**}$ as defined in proposition 4. From that point onwards the level of capital under IE lies below the PE equilibrium level and the difference converges to the constant gap described in proposition 2.

2.5 Different Steady States

So far the relationship between the level of income per capita and the effect of CI relies on the existence of absolute convergence. Now we would like to study how the effect of CI interacts with distortions affecting both the steady state and the transition to it from a low level of capital per capita. The most straightforward way of doing this is to allow $\hat{z}$ to vary, which makes the model exhibit conditional convergence under PE. Unlike ρ, changes in $\hat{z}$ affect output throughout the development process. But notice that all the results so far are unchanged. This is because Proposition 1 and the steady-state growth rate are independent from the constant $\hat{z}$. This implies that the dynamics of $\tilde{V}$ and $\tilde{V}^{d}$ are independent of $\hat{z}$ as well, and hence we still have the threshold level of capital determining the point at which the constraint becomes binding going forward. But a lower $\hat{z}$ generates a slowdown in the growth process outside the steady state. The reduction in the expected return generated by a lower productivity reduces savings, lowering capital accumulation and the growth rate of output and wages. Therefore, the relative continuation utility of honoring the contract, $\tilde{V}$, falls, while the relative continuation utility of defaulting, $\tilde{V}^{d}$, rises for any value of the stock of capital. It follows that this type of friction reduces the level of output at which the IC constraint becomes binding.

The left-hand panel of Figure 1 illustrates this pattern. The solid lines show the paths for capital per effective unit of labor in the IE and PE equilibriums for a certain economy with $\hat{z} = z_h$. The corresponding cut-off level is denoted by $K(z_h)^{**}$. The dashed lines illustrate the same paths for an economy with a lower level of $\hat{z} = z_l$. Capital is lower throughout the development process.

In this case, since growth and the incentives to honor contracts are lower, inefficient CI start hinder development at a lower level of capital, denoted by $K(z_l)^{**}$.[18]

If we interpret the parameter $\hat{z}$ more broadly, as an indicator of the different distortions affecting the steady state of the economy, then its inclusion in the model illustrates that the effects of CI on output per capita depend on the distance from the steady state, and not necessarily on the level of the country's development of the country. Only when $\hat{z}$ is the same among countries will differences in the value of income per capita contain all the necessary information to predict the effect of CI, because in that case these differences will be due to transitional dynamics and not to these distortions. This needs to be accounted for in the empirical exercise.

3 Collateral Constraints

In this section we extend the model to allow entrepreneurs to overcome the severity of financial constraints with internal funds. To do this we introduce ex-post heterogeneity through idiosyncratic shocks, both for workers and entrepreneurs, keeping the market structure as in the baseline model.[19] Thereby they accumulate assets as a precaution to future bad idiosyncratic shocks as in Aiyagari (1994) and Huggett (1997). In this framework we obtain an endogenous distribution of assets, which is stationary in steady state, and compute the transition generated by an unexpected jump in aggregate productivity from one stationary equilibrium to the other. This numerical exercise allows us to quantitatively evaluate the predictions described in the last section in a more general environment.

In principle, asset accumulation may offset the main predictions if these can be used as collateral as in the papers on limited enforceability and development revised in the literature review. In these papers, because agents lose their assets at the time of default, wealthier entrepreneurs have less incentives to default. Hence, if entrepreneurs increase their asset holdings fast enough during the transition, then the incentives to default may decrease, despite the effects of lower growth rates through the mechanism derived in the previous section. Before generalizing the model, we can see this by modifying the IC constraint defined in the previous section, allowing risk-neutral entrepreneurs to maintain a certain level of assets when they have not defaulted. In this case the current benefits from defaulting would be $\rho(r(\hat{K})+\delta)\hat{k}-(1+r(\hat{K}))\hat{a}$, where $\hat{a}$ are the (transformed) stock of assets the entrepreneur loses at the defaulting moment. Hence (3) and (4) become, respectively,

$$IC(\hat{K})=\beta\left[\gamma V(\hat{K}')-\rho\bar{\gamma}V^d(\hat{k};\hat{K}')\right]-\rho(r(\hat{K})+\delta)\hat{k}+(1+r(\hat{K}))\hat{a}\geq 0,$$

and

$$IC^{\mathrm{PE}}(\hat{K})=\beta\left[\gamma\tilde{V}(\hat{K},\hat{K}')-\rho\bar{\gamma}\tilde{V}^d(\hat{K},\hat{K}')\right]-\rho\alpha+\frac{(1+r(\hat{K}))\hat{A}}{\hat{Y}}\geq 0,$$

where $\hat{A}$ is the stock of assets owned by entrepreneurs and where $V(\hat{K}')$ and $\tilde{V}(\hat{K},\hat{K}')$ should be appropriately modified to include entrepreneurs' savings decisions.[20]

[18] We omit in this figure the fact that output is initially higher under worse CI.

[19] With homogeneous agents and no risk, as in the baseline model, the distribution of assets between workers and entrepreneurs along the balanced growth path could take any value under PE. Under IE, since entrepreneurs would get a higher return on their savings, they would accumulate all the assets in the economy in the long run, which is unrealistic, and the same would happen if only entrepreneurs face idiosyncratic risks.

[20] In particular, $\tilde{V}(\hat{K},\hat{K}')=(1-\alpha-\upsilon)(\hat{Y}'/\hat{Y})+((1+r(\hat{K}'))A'-A'')/\hat{Y}+\beta\gamma\tilde{V}(\hat{K},\hat{K}'')$.

Focusing on the collateral constraint this last expression shows that, beside expected growth, it is the ratio of entrepreneurs' assets to output, and not the total stock of the first, what matters for the binding dynamics of the constraint. If this ratio grows fast during the transition the right-hand side of the IC constraint falls, and hence it may become less binding as the economy approaches its steady state. This distinction is relevant in a growth context, since there is no obvious relationship between the ratio of entrepreneurs' assets to output, and the level of income per capita. The main reason why we may observe in equilibrium a high growth in this ratio would be because entrepreneurs may want to alleviate financial frictions by accumulating collateral. But this behavior would be caused by the constraint when it binds, and what the model predicts is that this doesn't happen at certain points during the transition. In this context we would observe high growth of this ratio only when the restriction becomes binding, which is relatively close to the steady state according to the model. Moreover, since under the optimal savings decision entrepreneurs are constrained the most in the stationary equilibrium, the incentives to accumulate assets would be strongest at that point. And given that previous work, described in the literature review, has shown that this would not suffice to overcome financial frictions in the long run, it follows that neither would it be during the transition. Therefore, under this reasoning, the accumulation of assets would not generally invalidate the mechanism proposed. In this section we solve numerically a general model to see if this is truly the case.

3.1 The Economic Environment

There is a continuum of entrepreneurs and workers of mass n and $1-n$, respectively. Entrepreneurs face two types of shocks. First, they can die with probability $1-q$ each period, independently of their wealth. A fraction $1-q$ of entrepreneurs are born every period, and start their life with zero assets. This is the simplest way to introduce a finite life span or a change in status in the standard model. Second, entrepreneurs face idiosyncratic shocks to their productivity level, which we denote by ϵ. This variable follows a Markov process with transition probability $\pi_\epsilon(\epsilon'/\epsilon)$. Adding this risk allows us to obtain a well defined stationary distribution of assets. Unlike in the previous section entrepreneurs are risk averse, so $\sigma^e = \sigma$, and accumulate assets a subject to a borrowing limit $a \geq 0$.[21] Workers accumulate assets subject to the same borrowing constraint and face idiosyncratic shocks to their labor productivity, which we denote by η. These follow a Markov process with transition probabilities $\pi_\eta(\eta'/\eta)$. As in the case of entrepreneurs, this allows us to obtain a well defined asset distribution for workers.

The cross-section distribution over individual state variables is $\Phi_e(a, \epsilon)$ and $\Phi_w(a, \eta)$ for entrepreneurs and workers respectively. Economy-wide productivity is deterministic and given by z as before, but now it doesn't grow, so we don't need to express the variables in transformed form.[22] Now we can define agents' problems. Defining $\Phi = \{\Phi_e, \Phi_w\}$, with law of motion $\Phi' = \mathrm{P}(\Phi)$, which

[21] Since there is not an option to become workers, entrepreneurs' income may become too low when facing a binding borrowing constraint if they don't have a different source of income. It is then assumed that entrepreneurs get an extra income, which we set as a very low fraction of the market wage. Since this does not affect the quantitative results we omit it in the description of the model.

[22] As shown in the baseline model, aggregate TFP growth complicates the exposition since the model needs to be transformed to work with a stationary model, but it is not relevant for the main results.

is given for all agents, the new problem for entrepreneurs is

$$V(a,\epsilon;\Phi) = \max_{c,a'\geq 0,k} \left\{ u(c) + q\beta \sum_{\epsilon'} \pi(\epsilon'/\epsilon) V(a',\epsilon';\Phi') \right\}$$

$$st. \quad c + a' = \Pi(k,\epsilon;\Phi) + a(1+r(\Phi)),$$

where

$$\Pi(k,\epsilon;\Phi) = \max_{n} \left\{ z\epsilon k^{\alpha} n^{\upsilon} - w(\Phi)n - (r(\Phi)+\delta)k \right\},$$

and subject to the IC constraint

$$V(a,\epsilon;\Phi) \geq \rho V^{d}(k,\epsilon;\Phi) + (1-\rho)H(\epsilon), \tag{5}$$

where H is the discounted utility for the agent that is caught after defaulting.

If not caught after defaulting, agents lose their assets and their access to financial markets, keep the stock of capital, and decide what fraction of their income to consume or reinvest in the firm. Hence, the value of default in this case is given by

$$V^{d}(k,\epsilon;\Phi) = \max_{c,k'} \left\{ u(c) + q\beta \sum_{\epsilon'} \pi(\epsilon'/\epsilon) V^{d}(k',\epsilon';\Phi') \right\}$$

$$st. \quad c + k' = \Pi^{d}(k,\epsilon;\Phi) + (1-\delta)k,$$

where

$$\Pi^{d}(k,\epsilon;\Phi) = \max_{n} \left\{ z\epsilon k^{\alpha} n^{\upsilon} - w(\Phi)n \right\}.$$

Therefore, unlike the entrepreneur that has access to workers' savings, investment is decided before the idiosyncratic shock is realized. Although not modeled here explicitly, this captures the ability of financial markets to allocate capital among projects with different rates of return. Since it is risky to invest, capital will be lower for defaulting entrepreneurs relative to those that haven't defaulted. Then, together with the costs of postponing consumption to achieve the first best level of capital, higher risk is behind the fact that defaulters cannot take full advantage of production opportunities.

To complete the model we present the dynamic problem for the worker, which is the standard in this type of models, and the equilibrium definition in Appendix A.

To better illustrate the effects from financial exclusion after default we compare below the numerical results with the ones obtained when the dynamic constraint (5) is replaced by the following static constraint,

$$k \geq (1+\lambda)a, \tag{6}$$

where $\lambda > 0$. This constraint, commonly used in the literature (e.g Buera and Shin, 2013; Moll, 2014), captures the idea that credit to entrepreneurs is limited by the amount of assets that can be pledged as collateral. Since this mechanism is already captured in constraint (5), different outcomes are mainly given by future exclusion from financial markets, the key feature behind the main result obtained in the last section.

3.2 Numerical Results

To analyze transitional dynamics in this economy, it is necessary to fix the initial state, which now is the distribution of individual states instead of just the aggregate level of capital. We use as a starting state the endogenous stationary distribution consistent with a value of z that is lower than the value for which we compute the transition and the stationary equilibrium to which this converges. Therefore the experiment is equivalent to an unexpected and permanent change in z, for which we need not keep track of the distribution as a state. Because the stationary distribution depends on ρ, unlike the illustration of the baseline model in the left panel of Figure 1, the initial level of capital differs between the PE and IE equilibriums. We choose parameter values that are in line with previous literature. Below we implement a sensitivity analysis with respect to some of them.[23]

The paths for capital for the PE and IE equilibriums are depicted in the left-hand panel of Figure 2. The transition to a new steady state, generated by the change in z, occurs in period $t = 0$. In the first years of the transition, the gap between the two economies is relatively small, and only after some years it starts to widen until it remains constant in the new steady state. To see this more clearly, the solid line in the right-hand panel of Figure 2 shows the gap between the two economies. The difference observed in the initial stationary equilibrium is significantly reduced in the first years of the transition. Thereafter, as growth slows down due to diminishing returns, the gap widens until it converges to the new steady state with a similar size than the one observed before the transition. Hence, we can see that the mechanism proposed in the last section is relevant in this new environment as output costs due to inefficient CI vary significantly with the distance of the economy to its steady state. These costs are relatively less important in the first years of the transition due to the self-enforcement effects of high growth rates. The dashed line in Figure 2 shows the gap between the PE equilibrium and the path obtained when the static constraint (6) replaces (5). In this case we choose a value of λ that generates the steady-state capital gap obtained in the IE equilibrium. It can be seen that the gap widens in the initial years of the transition, validating that the main result comes from exclusion from financial markets after default.

To illustrate how the mechanism works in this new environment Figure 3 shows the binding pattern of the IC constraint during the IE transition. In the left-hand panel we show the minimum level of assets, as a fraction of aggregate capital, necessary to be able to finance the efficient level of capital through workers' credit for different levels of the idiosyncratic shock, ϵ (except for the worst realization, when the constraint never binds). This level of assets drops in the first year of the transition, due to the fall of default incentives, for every level of ϵ, meaning that entrepreneurs

[23] Since we are interested in the timing of the effects of CI on output, we parameterize the model to obtain output costs from IE in the stationary equilibrium (i.e. $\Omega < 1$ if we were in the model without collateral), and explore the pattern of these costs during the transition. To obtain this we use a value for q of 0.99. We follow Buera and Shin (2013), who apply a similar model to long-run growth transitions, to set $1-\alpha-\upsilon = 0.21$, $\alpha = 0.33\times(1-0.21) = 0.26$, and $\beta = 0.92$. Their specification for idiosyncratic shocks is different from ours, so we follow Moll (2014) and Asker et al. (2014) and set the persistence for the idiosyncratic shocks (AR(1) coefficient) to 0.85. The standard deviation of the shocks are chosen such that both entrepreneurs and workers face the same risk in the PE equilibrium (i.e. same variance for wages and profits), and to obtain an equilibrium interest rate in the IE stationary equilibrium equal to 4%. This means standard deviations of 0.12 and 0.55 for the logs of ϵ and η, respectively. Then n is set such that the average wage is equal to average profits under PE, which implies $n = 0.285$. For the penalty for defaulting $H(\epsilon)$ we assume that in the first period the entrepreneur can only consume 15% of the equilibrium wage, and from then on he becomes a worker starting without assets. We set $\rho = 0.75$ and use standard values $\delta = 0.06$ and $\sigma = 2$.

are able to finance a larger fraction of investment with external funds. Since collateral is not as important as in the stationary equilibrium, the entrepreneurs' incentives to accumulate assets are not as strong during that stage. After the initial drop these ratios converge to levels close to those observed in the initial steady state, showing that the change in the incentives to default is only transitory because of the transitory nature of high growth rates. As a consequence, the fraction of entrepreneurs that are constrained (i.e. that are not able to invest the efficient level), decreases as well in the initial years after the transition. This is shown in the right-hand panel of Figure 3 for each productivity level (except again for the worst realization). Similarly to the other indicators, this fraction converges to the levels observed in the initial steady state.

Since the severity of financial frictions changes endogenously along the transition the intensity of capital misallocation between entrepreneurs with different productivities should vary as well. In Figure 4 we plot the fraction of the steady-state gap that persists during the transition for both capital and output. Interestingly, the gap for output decreases more than the one for capital in the first periods, a feature that is reverted as the economy converges to the steady-state. This is explained by the fact that in the first years of the transition the endogenous relaxation of financial constraints leads to an improvement in the allocation of capital. In fact, in the stationary equilibrium capital misallocation accounts for 27% of the output gap with respect to the PE equilibrium, a value that decreases to less than 4% in the first years of the transition.

To finish the analysis of the extended model we pursue a sensitivity analysis with respect to key parameters; those governing productivity shocks, the span of control, and the strength of CI. Since the value of these parameters affect the size of the capital gap in steady-state we normalize the initial gap to zero in each case. Results are presented in Figure 5. The upper panel shows variations to the processes of productivity shocks. In the left-hand panel we modify the AR(1) coefficient, which is 0.85 in the baseline calibration. We show how results change when using a higher value of 0.95 (dotted line), and a lower value of 0.75 (dashed line). The reduction in the capital gap seems to be larger and longer-lasting for higher persistence, although the size of the effects are relatively small. Then we vary the variance of the productivity processes. We compare the baseline with respect to a process with 10% higher variance (dotted line) and 10% lower variance (dashed line). In the upper-right-hand panel we can see that the reduction of the gap is increasing in the volatility of productivity shocks. The same happens with the span of control. This parameter is 0.21 in the baseline calibration, the value used by Buera and Shin (2013). In the lower-left-hand panel of Figure 5 we show results when solving the model with a value of 0.18 (dashed line) and 0.24 (dotted line).[24] The reduction in the gap is relatively small when using a low value for this parameter. It is worth noting however that the steady-state gap is also reduced, and that almost 20% of this gap is closed in the first years (compared to 35% in the baseline calibration). Finally we analyze changes to the quality of CI. Using values of 0.65 (dashed line) and 0.85 (dotted line) for ρ, which in the baseline calibration is 0.75, we can see that the reduction of the gap is increasing in this parameter (i.e. decreasing in the quality of CI), as expected.

[24] In this case n and the standard deviation of ϵ are modified to equalize average wages and profits under PE, and to obtain the variances for these variables used in the baseline calibration.

4 The Evidence

The previous section shows that the size of the effect of the quality of CI on income per capita will be stronger the closer the economy is to its steady-state. In this section we explore empirically this prediction for a cross-section of countries. To do this we exploit the following idea. Suppose we are able to find conditional convergence, which means we find the determinants of the steady-state level of output. Hence if we control for steady-state determinants different from CI we would obtain, according to the model, paths for output similar to the ones depicted in the left-hand panel of Figure 1. Note that if the prediction of the model is true the effect of CI will depend on the level of output of the economy. If we measure the effect below K^{**} then we shouldn't find an effect. If we do so for levels above that threshold we should find and effect. And as we move to the right, measuring output at a higher levels, we should find larger effects. This variation can be used to explore the prediction empirically, but because output is the endogenous variable we cannot use it to capture the stage at which the country is along the transition. We discuss how we exploit this idea below, before showing the econometric results.

4.1 Empirical Strategy

Here we explain how the mechanism proposed in the last sections can be tested using the structures of previous empirical work. We start with a simple regression equation. We define $Y_{t,i}$ as GDP per capita in country i and period t (if we consider the model in section 2 we assume $\mu = 1$ so for any variable x, $\hat{x} = x$). Suppose ρ_i captures the exogenous component of the nature of CI, and z_i any other distortion affecting the economy in country i, as defined in the model. The first specification takes the following form,

$$\log Y_{t,i} = \alpha_0 + \alpha_1 z_i + \alpha_2 (1 - \rho_i) + \epsilon_i \tag{7}$$

Acemoglu and Johnson (2005) estimate this equation using an indicator of PRI for z_i, for a large sample of countries, and find that α_1 is positive and significant, while α_2 is not significant.

Because in the model the steady-state level of output per capita is determined by z_i and ρ_i, we can estimate the regressions implemented by Barro (1991) and Barro and Sala-i-Martin (1992, 2004) as,

$$\log Y_{t,i} = \beta_0 + \beta_1 z_i + \beta_2 (1 - \rho_i) + \beta_3 \log Y_{t-T,i} + \zeta_i \tag{8}$$

where $T > 0$, and conditional convergence implies $\beta_3 < 1$. This means that, controlling for the steady-state value of output per capita, or z_i and ρ_i, transitional dynamics imply that low income countries grow faster than high income countries. Once we have an unambiguous and negative relationship between the level of income per capita at $t - T$ and the subsequent growth of the economy between $t - T$ and t, we have, according to the model, an unambiguous and positive relationship between the level of income at $t - T$ and the effect of ρ_i on the subsequent growth of the economy. Therefore we introduce an interaction term between ρ_i and $Y_{t-T,i}$ to estimate the following regression,

$$\log Y_{t,i} = \gamma_0 + \gamma_1 z_i + \gamma_2 (1 - \rho_i) + \gamma_3 \log Y_{t-T,i} + \gamma_4 (1 - \rho_i) \log Y_{t-T,i} + \upsilon_i \tag{9}$$

where $\partial \log Y_{t,i} / \partial (1 - \rho_i) = \gamma_2 + \gamma_4 \log Y_{t-T,i}$ keeping z_i constant. The model predicts this derivative to be increasing in $Y_{t-T,i}$, as expected growth, conditional on the steady-state level of income per capita, is decreasing in this variable. Hence, we expect $\gamma_4 > 0$.

To better illustrate that this relationship is indeed predicted by the model, we construct data by numerically solving the deterministic model presented in section 2 and estimate regressions (7), (8), and (9).[25] For the two parameters that vary across countries $-\rho_i$ and z_i- we assume them to be *iid*.[26] Once we have chosen all the parameters we solve the model for N different pairs (ρ_i, z_i), and obtain $\{Y_{0,i}, ..., Y_i^{ss}\}_{i=1}^N$, normalizing $K_{0,i} = \underline{K}\ \forall i$. As we do not know in which stage of the transition each country is, we assume they are stochastically distributed on a bounded set, i.e. $t_i \in [T, \bar{T}]$.[27] Since this defines the distribution of countries with respect to their steady states, which is key to finding a non-linear effect of CI, we choose a combination that generates a similar size for the coefficient of initial GDP per capita as the one we find later with real data. These values are $T = 7$ and $\bar{T} = 47$. After doing this we obtain $(Y_{t_i}, Y_{t_i-T}, \rho_i, z_i)_{i=1}^N$. For clarity we rewrite the subindexes as in the regressions above, so we have $(Y_{t,i}, Y_{t-T,i}, \rho_i, z_i)_{i=1}^N$, i.e. a cross-section with which we can estimate regressions (7), (8), and (9).

Results are presented in Table 1. In the first column we use as the dependent variable the value of output in steady state. We normalize the explanatory variables such that each coefficient is equal to 1. This is to make clear how the coefficients change when we do not restrict the countries to be in their steady states. This is done in column (2), where the dependent variable is now $\log Y_t$. While the coefficient of z remains constant, the coefficient of $(1-\rho)$ falls about 50%. This illustrates how the mechanism in the model explains the small effect of CI when estimated in a cross-country regression, even when modifying contracts is costly. In the presence of measurement problems or small but significant costs of changing contracts, it obstructs the finding of a significant effect. In column (3) initial GDP is included and the coefficient is lower than one, capturing conditional convergence. Results from running specification (9) are presented in column (4), where we can see that, as expected, the interaction term has a significant and positive effect on GDP per capita, i.e. $\gamma_4 > 0$. To show that this is indeed an implication of the main mechanism in the model in column (5) we include an interaction term but with z and obtain the opposite result. This validates the test to the theory proposed above.

4.2 Identification and Data

The numerical example provided in the last subsection guides the empirical estimations.[28] Since we want to confirm the results by Acemoglu and Johnson (2005), and, more importantly, because of the close relationship between CI and PRI, we assume that z is well captured by this second type of institutions. Hence, we follow very closely the identification strategy proposed by Acemoglu and Johnson (2005), which is based on the observation that some exogenous variables are significant explaining one type of institutions but not the other.

[25] We pick $\delta = 0.06$, $\beta = 0.96$, and $\sigma = 2$ $\alpha = 0.4$ and $\upsilon = 0.58$. Since this model is more restrictive the range of parameters to find output costs from frictions is smaller. Although in principle parameter values may influence the finding of a non-linear effect for CI, ultimately it is how we pick the period for the data used in the regressions the most important part, as this determines the fraction of countries outside their steady states. Below we explain how this period is chosen.

[26] Specifically, we use $x_i^1 \sim N(0,1)$ and $x_i^2 \sim N(0,1)$ to define $\rho_i = 1/(1+exp(x_i^1))$ and $z_i = 1/(1+exp(x_i^2))$.

[27] Specifically, we draw N values $x_i \sim N(0,1)$ to define $t_i = T + (\bar{T} - T)/(1+exp(x_i))$.

[28] The model is very stylized and in practice the convergence is not smooth. For a given value of z, policies, the proximate cause of growth, may be changing gradually, and hence countries may experience periods of high and low growth during their transitions. The existence of conditional convergence shows however that on average poor countries with relatively good institutions grow faster, and hence CI would have smaller effects on growth.

The literature linking institutions and long-run growth is large, as described above. The main empirical problem of these studies is that available measures of institutional quality are outcomes and therefore they are affected by actual economic conditions, making causal relationships difficult to identify. To overcome this problem past studies have used instruments to capture the exogenous component of the quality of institutions. These instruments are based on the idea that the nature of the institutional framework is highly persistent and was mainly shaped by the influence of European countries. In the case of CI, it has been widely documented that the main exogenous variation comes from differences in legal traditions spread by European countries through conquest, imitation, and colonization (Levine, 2005; La Porta et al., 2008). These traditions differ importantly in their ability to adapt to evolving economic conditions (Beck et al., 2003; Levine, 2005). On the other hand, Acemoglu et al. (2002) propose a measure of initial endowments as instruments. They show that areas that were relatively rich in 1500 are now relatively poor countries. Their explanation is that in poorer areas Europeans established institutions of private property that favored long-run growth, while in richer areas they established extractive institutions, which discourage investment and economic development (see also Engerman and Sokoloff, 2002). Therefore, indicators related to initial endowments are good instruments to capture the exogenous component of PRI quality. In particular, Acemoglu et al. (2002) show that urbanization and population density in 1500 strongly reflect these determinants.

Acemoglu and Johnson (2005) take advantage of the strong link between the quality of CI and legal origin on the one hand, and the quality of PRI and initial endowments on the other, to identify the exogenous component of each of these variables, and further to unbundle their effects on income per capita and financial development. The same strategy is used in this paper to identify the effect of these institutions on income per capita for a sample of former colonies since the theory outlined by Acemoglu et al. (2002) between initial endowments and institutional quality applies only to these countries. Accordingly legal origin is used for capturing the exogenous component of CI, and population density in 1500 is used for capturing the exogenous component of PRI.

But once the link among these variables has been verified, there is an alternative to TSLS to perform the tests. In particular, we can use the instruments as explanatory variables for GDP per capita today and interpret the coefficients as an approximation to the effect of the corresponding institution. Although it is not possible to reject that there may be additional channels explaining the estimated coefficients, the advantages of doing this is that we do not need institutional measures, which are imperfectly constructed and endogenous, and that only a much simpler OLS regression needs to be estimated. Acemoglu et al. (2002) find a negative and statistically significant effect of initial endowments on today's GDP per capita, while other possible explanatory variables, including legal origin, do not have a statistically significant effect on today's GDP per capita. Consequently we use TSLS, using instruments and institutional indicators, as well as OLS, using only the instruments, to perform the test described above.

The primary source for legal origin is Djankov et al. (2008), who focus on the legal origin of a country's bankruptcy laws. If not available, we use the data from Djankov et al. (2003). The source for population density in 1500 is Acemoglu et al. (2002). Regarding institutional indicators, in the case of CI Acemoglu and Johnson (2005) use the index of legal formalism constructed by Djankov et al. (2003), which is a measure of the number of legal proceedings arising from the collection of a bounced check. This index does not measure costs explicitly, although the authors show that it is correlated with the delay in the resolution of disputes. More recently Djankov et al. (2008) construct an indicator of contract enforcement, which explicitly includes most of the costs of debt enforcement which deter creditors' legal actions against fraudsters. Since it reflects more closely the features of CI relevant for the main prediction of the model, we use this index as our preferred measure. Doing this however reduces importantly the number of countries available for the regressions relative to the index of legal formalism. Therefore we show the results with both of these indexes (in the case of legal formalism we use its negative value so we expect a positive interaction). Hence, in the case of legal formalism we claim that the number of procedures is a determinant of litigation costs and of the incentives by lenders to initiate a legal dispute. Using the legal formalism index also allows us to compare more closely our results with those obtained by Acemoglu and Johnson (2005). For PRI indicators, Acemoglu and Johnson (2005) use executive constraints (XC) from the Polity IV database as their preferred measure. This index measures explicitly how constrained the executive is in making arbitrary decisions.

The source for GDP per capita is Maddison (2008). For initial GDP per capita the year 1950 is used because it is the earliest for which data on GDP per capita for a large number of former colonies is available. As the dependent variable we use GDP per capita in 2006, and accordingly we use the 20-year average prior to that year for XC.

Although in the model the steady-state level of GDP per capita is solely determined by institutions, this is clearly a simplification. Indeed in some of our estimations we are not able to find conditional convergence when controlling only for our CI and PRI indicators. Introducing new variables to capture steady-state determinants may be problematic because we lack a known identification strategy to confront endogeneity problems and because most of the possible candidates are, at least in part, outcomes of institutions. To overcome these problems we use a proxy of initial human capital, an important determinant of the steady-state level of income per capita, and an exogenous variable in the context of Equation (8). We use data on secondary enrollment in 1960 as reported by Barro and Lee (1993). This year is the earliest period for which this indicator is available for a large number of former colonies.

Table 2 summarizes the data. In the upper panel we show the main statistics and in the lower panel the correlation matrix. Our two indicators of CI are strongly correlated, and the correlation between them and XC is, perhaps surprisingly, small, particularly in the case of contract enforcement. There is a high correlation between legal origin and our CI indicators on the one hand, and between XC and population density in 1500 on the other.[29]

4.3 Empirical Results

Table 3 shows the first-stage regressions. As expected, when not controlling for the initial level of income per capita (columns 1, 3, and 5), legal origin has a strong and significant effect on our CI indicators and a non-significant effect on XC, and in the case of population density we observe a strong and significant effect on XC but not on CI indicators. This confirms the identification strategy proposed by Acemoglu and Johnson (2005) since independent exogenous components are identified for each of the institutional indicators. Despite our smaller sample, our results are

[29]For a list of the countries included in the estimations and the data used see Appendix C. Angola is dropped from the sample because it is an outlier. There is no data for legal formalism and it has a contract enforcement index of 0.18, more than 4 standard deviations below the average, and almost 3 standard deviations below the second worst value. Our sample is smaller than the one used by Acemoglu and Johnson (2005), who have data for 60 countries. They report they use an update of Djankov et al. (2003), supplied privately, for legal formalism.

quantitatively very similar to theirs.[30]

When we include initial GDP per capita in columns (2), (4), and (6), our results for XC change. Now population density in 1500 is not significant, while initial GDP per capita is highly significant (column 6). Notice however that explaining CI indicators (columns 2 and 4), this last variable is not significant, while English origin remains highly significant. Hence, despite the fact that population density in 1500 becomes not significant explaining PRI, the logic behind the identification strategy proposed by Acemoglu and Johnson (2005) still holds; exogenous variables, now for the period following 1950, are highly significant in explaining one type of institutions but not the other. The shortcoming is that once we include initial GDP per capita as a regressor in the second stage, the coefficient of PRI is not valid anymore. Accordingly the interaction effect between PRI and initial GDP per capita is not informative. But with respect to our test, since we do not have a theory for PRI, this is not a relevant problem. In any case, notice that under the assumption that PRI capture variations in z, this result is in line with the main prediction of the model as, unlike ρ, this variable affects output throughout the development process.

The second-stage results, together with the OLS estimates using the instruments as explanatory variables, are presented in Table 4. In the first panel we use our preferred measure of CI, contract enforcement, and legal origin and population density in 1500 as instruments. In column (1) the results by Acemoglu and Johnson (2005) are confirmed: contract enforcement is not significant and the effect of XC is positive and significant. When controlling for GDP per capita (column 2) we can see that the coefficient on XC falls and becomes not significant. As already discussed, we cannot identify the exogenous component of PRI anymore so this result is not very informative besides the fact that PRI, unlike CI, had an effect on GDP per capita in 1950. The coefficient of initial GDP per capita is lower than one at a 5% significance level, confirming conditional convergence. The interaction term between the CI indicator and initial GDP per capita is included in column (3). It is positive, as expected, but not significant. However this doesn't imply that the effect is insignificant for all countries. To better illustrate the results, we present the effect of contract enforcement for the richest country in 1950 in our sample, on an additional row in Table 4. The coefficient is positive and significant at a 1% level of significance. The estimations imply that the effect of contract enforcement is positive and significant at a 10% level above a value for log GDP per capita in 1950 of 7.8, i.e. conditional on 20% of the richest countries in 1950. This does not necessarily imply that only this fraction of richest economies are affected by CI, as the same steady-state level is assumed in this case.

As noted above, although legal formalism is not as good as contract enforcement mapping the model into the data, using it as the CI indicator allows us to enlarge the sample and compare our results with those of Acemoglu and Johnson (2005). Panel 2 in Table 4 presents the results when this measure is used instead of contract enforcement. Again, despite our smaller sample, results without conditioning on initial GDP per capita are similar to those of Acemoglu and Johnson (2005). XC are very significant while legal formalism is not (column 4).[31] Again, after introducing GDP per capita in 1950 (column 5), the coefficient of XC falls and becomes not significant, while conditional convergence is significant at the 10% level. The inclusion of the interaction term in column (6) implies that legal formalism is positive and significant at a 10% level above a value for

[30] For legal formalism they report 1.79 (0.20) and -0.04 (0.06) as the coefficients (standard errors) for legal origin and population density, respectively, and 0.05 (0.43) and -0.4 (0.13) for XC. Their R^2s are 0.58 and 0.15, respectively.

[31] Acemoglu and Johnson (2005) report 0.002 (0.21) and 0.88 (0.27) as the coefficients (standard errors) of legal formalism and XC, respectively.

log GDP per capita in 1950 around 8.35, i.e. conditional on 14% of the richest countries in 1950.

Given the strength of the results in the first stage, it is possible to implement our tests using instruments as explanatory variables for GDP per capita today. This exercise may be useful to infer the effects of each type of institution without a specific, and probably imperfect, measure of their quality, although we cannot discard the influence of additional variables in the results. The last panel in Table 4 uses legal origin and population density in 1500 as explanatory variables. Confirming Acemoglu et al. (2002)'s results, the effect of legal origin on the unconditional level of GDP per capita is not significant when controlling for initial endowments (column 7). When including GDP per capita in 1950 in column (8), conditional convergence is significant at a 1% significance level. Finally, when the interaction term is included in column (9), we find a significant effect of legal origin in initially richer countries conditioning for steady states. The effect of CI becomes significant now at levels of GDP per capita shown by the richest 10% of the countries (above a value for log GDP per capita of 8.72) at that time at a 10% significance level.

Taking all the results together we can conclude that the evidence supports the main prediction of the model. The significant interaction we have estimated means that the effect of CI on GDP per capita during the last 60 years has been significant only in those economies that were relatively close to their steady states in 1950.

5 Conclusions

This paper studies the effect of CI on development. A growth model with endogenous financial frictions induced by imperfect enforceability of contracts is presented. The key assumption is that after defaulting, producers are unable to take full advantage of future production opportunities. This generates the main implication of the model: financial frictions are more important when expected growth is low, because in that case self-enforcement incentives are weak. As high quality CI reduce the benefits of defaulting, they are irrelevant when self-enforcement incentives are strong. But otherwise, low quality CI slow down capital accumulation and economy-wide TFP growth. After embedding these features into the standard neoclassical growth model the paper predicts that the effect of the quality of CI on GDP per capita depends on the distance between current output and its steady-state level. The closer the economy is to its steady state, the larger are the effects of the quality of CI on income per capita. The robustness of this prediction, which is the main contribution of the paper as it hasn't been identified before in the literature on finance and development, is explored in a more general model of heterogeneous agents and incomplete markets where entrepreneurs can save to overcome financial frictions.

Simulated data shows that this mechanism can contribute to the understanding of past evidence as finding a significant relationship between CI and income per capita is obscured by the fact that many countries are far from their steady states. We derive a testable implication of the model which is confirmed by the simulated data. In particular, after controlling for the steady-state level of output per capita, we should observe a positive interaction effect between CI and initial GDP per capita. The paper implements cross-country regressions to test this implication using the identification strategy proposed by Acemoglu and Johnson (2005). After confirming the results presented by these authors the empirical evidence is in line with the prediction of the model. The main finding is that the effect of CI on output per capita growth in the last 60 years is significant only for countries that were relatively close to their steady states in 1950.

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Appendix A

The Transformed Dynamic Programming Problems

Given that $\hat{z}$ is constant, the aggregate state of the world is now described by $\hat{K}$. First it is necessary to compute the steady-state growth of consumption for an entrepreneur that has defaulted in a previous period t^*. Income for this entrepreneur in period $t > t^*$ is $z_t \bar{k}_t^\alpha n_t^\upsilon - w_t n_t$, which, using the demand for labor, is

$$(1-\upsilon)\left[\frac{\upsilon^\upsilon z_t \bar{k}_t^\alpha}{w_t^\upsilon}\right]^{\frac{1}{1-\upsilon}}$$

using $\bar{k}_t = (1-\delta)\bar{k}_{t-1}$, and the asymptotic growth rate of w, the asymptotic growth rate of income and, given $\sigma^e = 0$, of consumption, will be

$$\gamma^d = \gamma^{\frac{1-\alpha-\upsilon}{1-\upsilon}}(1-\delta)^{\frac{\alpha}{1-\upsilon}}$$

Now $\hat{c}_t^d = c_t^d/(\gamma)^{t^*}(\gamma^d)^{t-t^*}$ can be defined, which will be constant in steady-state. For $t > t^*$, $u(c^d) = \gamma^{t^*}(\gamma^d)^{t-t^*}u(\hat{c}^d)$, and $V_{jt+1}^d(\bar{k}; K') = \gamma^{t^*}(\gamma^d)^{t-t^*}V_{jt+1}^d(\hat{k}; \hat{K}')$. For non-defaulting entrepreneurs and consumers, consumption of good j grows at the constant rate γ as noted in the text. Then we define $\hat{c}_t^i = c_t^i/\gamma^t$ for $i = c, e$, which will be constant in steady-state, and $u(c^i) = \gamma^t u(\hat{c}^i)$. The last step is to transform the budget constraints and the market clearing conditions. Notice that in every case the LHS and RHS grow at the same rate in steady state so transforming them is simple. When savings are included however we have $b_{t+1}/\gamma^t = \gamma b_{t+1}/\gamma^{t+1} = \gamma\hat{b}_{t+1}$. This adjustment is made to the budget constraint and the market clearing conditions. Now the transformed problem facing the entrepreneur is

$$V(\hat{K}) = \max_{\hat{k},n}\left\{u(\hat{c}) + \beta\gamma V(\hat{K})\right\}$$

$$st. \quad \hat{c} = \hat{y} - \hat{w}(\hat{K}) - (r(\hat{K}) + \delta)\hat{k}$$

$$u(\hat{c}) + \beta\gamma V(\hat{K}) \geq \rho\left[u(\hat{y} - \hat{w}(\hat{K})) + \beta\bar{\gamma}V^d(\hat{k}; \hat{K}')\right] + (1-\rho)u(\hat{c}),$$

where

$$V^d(\hat{k}; \hat{K}') = \max_{n'}\left\{u(\hat{y}' - \hat{w}'(\hat{K}')) + \beta\bar{\gamma}V^d(\hat{k}; \hat{K}'')\right\}$$

And the transformed consumers' problem will be

$$U(\hat{b}; \hat{K}) = \max_{\hat{c},\hat{b}'}\left\{\frac{\hat{c}^{1-\sigma}-1}{1-\sigma} + \beta\gamma^{1-\sigma}U(\hat{b}'; \hat{K}')\right\}$$

$$st. \quad \hat{c} + \gamma\hat{b}' = \hat{w}(\hat{K}) + \hat{b}(1 + r(\hat{K}))$$

Finally, the transformed market clearing condition is,

$$\hat{C} = \sum_i \hat{c}^i = \hat{Y} + (1-\delta)\hat{K} - \gamma\hat{K}'$$

Workers' Problem and Equilibrium in the Model of Collateral Constraints

The dynamic problem for the worker is

$$U(a, \eta; \Phi) = \max_{c, a' \geq 0}\left\{u(c) + \beta\sum_{\eta'}\pi(\eta'/\eta)U(a', \eta'; \Phi)\right\}$$

$$st. \quad c + a' = \eta w(\Phi) + a(1 + r(\Phi))$$

Definition 2. *A stationary competitive equilibrium, given a value for z, is a set of decision functions $c^e = \mathrm{C}^e(a,\epsilon;\Phi)$, $a'^e = \mathrm{A}^e(a,\epsilon;\Phi)$, $n = \mathrm{N}(a,\epsilon;\Phi)$, $k = \mathrm{K}(a,\epsilon;\Phi)$, $c^d = \mathrm{C}^d(k,\epsilon;\Phi)$, $k'^d = \mathrm{K}^d(k,\epsilon;\Phi)$, $n^d = \mathrm{N}^d(k,\epsilon;\Phi)$, $c^w = \mathrm{C}^w(a,\eta;\Phi)$, and $a'^w = \mathrm{A}^w(a,\eta;\Phi)$, a set of pricing functions $w = \mathrm{W}(\Phi)$ and $r = \mathrm{R}(\Phi)$, and an aggregate law of motion for distributions $\Phi' = \mathrm{P}(\Phi)$, such that,*

1. *Entrepreneurs solve their dynamic programming problem, given $\mathrm{P}(\cdot)$, $\mathrm{W}(\cdot)$ and $\mathrm{R}(\cdot)$, with the equilibrium solution satisfying $c^e = \mathrm{C}^e(a,\epsilon;\Phi)$, $a'^e = \mathrm{A}^e(a,\epsilon;\Phi)$, $n = \mathrm{N}(a,\epsilon;\Phi)$, $k = \mathrm{K}(a,\epsilon;\Phi)$, $c^d = \mathrm{C}^d(k,\epsilon;\Phi)$, $k'^d = \mathrm{K}^d(k,\epsilon;\Phi)$, and $n^d = \mathrm{N}^d(k,\epsilon;\Phi)$.*

2. *Workers solve their dynamic programming problem, given $\mathrm{P}(\cdot)$, $\mathrm{W}(\cdot)$ and $\mathrm{R}(\cdot)$, with the equilibrium solution satisfying $c^w = \mathrm{C}^w(a,\eta;\Phi)$, and $a'^w = \mathrm{A}^w(a,\eta;\Phi)$.*

3. *For all Φ, the following market clearing conditions hold,*

$$\begin{aligned}
(1-q)\int a'^e(a,\epsilon;\Phi)d\Phi_e + \int a'^w(a,\eta;\Phi)d\Phi_w \equiv A' &= K' \equiv \int k(a',\epsilon';\Phi')d\Phi'_e \\
\int n(a,\epsilon;\Phi)d\Phi_e &= \int \eta d\Phi_w \\
\int c^e(a,\epsilon;\Phi)d\Phi_e + \int c^w(a,\eta;\Phi)d\Phi_w + K' &= \int y(a,\epsilon;\Phi)d\Phi_e + (1-\delta)K
\end{aligned}$$

4. *The aggregate law of motion P is generated by the exogenous Markov processes π_ϵ and π_η, and the policy functions $a'^e(a,\epsilon;\Phi)$ and $a'^w(a,\eta;\Phi)$.*

To obtain a stationary equilibrium we impose in this definition a stationary distribution $\Phi^* = \mathrm{P}(\Phi^*)$.

Appendix B: Proofs

Proof of Proposition 1

It is straightforward to see that the competitive equilibrium is efficient when $\rho = 0$, in the sense that it coincides to the solution to the following central planner recursive problem,

$$W(\hat{k}) = \max_{\hat{k}',\hat{c}^c,\hat{c}^{e_j}} \lambda u(\hat{c}^c) + (1-\lambda)u(\hat{c}^e) + \beta\gamma^{1-\sigma}W(\hat{k}')$$

subject to (1) and (2), and where $0 < \lambda < 1$ is set arbitrarily but it is not a choice variable. The static problem for the central planner consists on the allocation of output net of savings between consumers and entrepreneurs. The solution to the dynamic problem is described by the policy function $\hat{k}' = g(\hat{k})$. The FOC and the envelope condition are the following,

$$\gamma u_{\hat{c}}(\hat{k},\hat{k}') = \beta\gamma^{1-\sigma}W_{\hat{k}'}(\hat{k}') \tag{10}$$

$$W_{\hat{k}}(\hat{k}) = u_{\hat{c}}(\hat{k},\hat{k}')\left(\alpha\hat{z}\hat{k}^{\alpha-1} + (1-\delta)\right) \tag{11}$$

where it is clear that $W(\hat{k})$ is increasing and concave on $\hat{k}$. Another property of this problem is that $g(\hat{k})$ is increasing on $\hat{k}$. To see this suppose that it is not true and $g(\hat{k})$ is decreasing on $\hat{k}$. Then $\exists\ \hat{k}_t,\ \hat{k}_\tau$ such that $\hat{k}_\tau > \hat{k}_t$ and $g(\hat{k}_\tau) < g(\hat{k}_t)$. Because $W_{\hat{k}}(\hat{k})$ is decreasing on $\hat{k}$ (by concavity and monotonicity of W), we have $W_{g(\hat{k}_\tau)}(g(\hat{k}_\tau)) > W_{g(\hat{k}_t)}(g(\hat{k}_t))$. This and (10) imply $u(\hat{k}_\tau, g(\hat{k}'_\tau)) < u(\hat{k}_t, g(\hat{k}'_t))$ and so $g(\hat{k}_t) > g(\hat{k}_\tau)$, which is a contradiction. Now it is possible to show that $d\hat{k} > 0$, $\forall\hat{k} < \hat{k}^{\text{ss}}$. Suppose that $\hat{k}^*$ and $\hat{k}'^*$ are part of the solution sequence, with $\hat{k}'^* = g(\hat{k}^*)$. By concavity of W we know $(\hat{k}^* - \hat{k}'^*)\left(W_{\hat{k}} - W_{\hat{k}'}\right) \leq 0$. Using (10) and

(11), this expression becomes $(\hat{k}^* - \hat{k}'^*)\left(\alpha\hat{z}\hat{k}^{\alpha-1} + (1-\delta) - \gamma^\sigma/\beta\right) \leq 0$. Notice that in steady-state (10) and (11) imply

$$\alpha\hat{z}(\hat{k}^{\text{ss}})^{\alpha-1} + (1-\delta) = \frac{\gamma^\sigma}{\beta} \tag{12}$$

so we have $(\hat{k}^* - \hat{k}'^*)\left(\hat{k}^{\alpha-1} - (\hat{k}^{\text{ss}})^{\alpha-1}\right) \leq 0$. Therefore, if $\hat{k} < \hat{k}^{\text{ss}}$, then $\hat{k} < \hat{k}'$, and so capital increases during the transition. From (1) we know output is also increasing, and so entrepreneurs' consumption is increasing as well. From the FOC for the entrepreneurs' problem, $\alpha\hat{z}^{1-\alpha}\hat{k}^{\alpha-1} = r + \delta$ and $\upsilon\hat{z}^{1-\alpha}\hat{k}^{\alpha} = \hat{w}$, we know that r is decreasing and $\hat{w}$ is increasing during the transition. These equations also imply that if the growth rate of capital decreases during the transition the same happens with output, entrepreneurs' consumption, and wages, and the opposite happens with the interest rate. Using the FOC for the consumer's problem, and (12) we get,

$$\left(\frac{\hat{c}^c}{\hat{c}'^c}\right)^{-\sigma} = \frac{\beta}{\gamma^\sigma}(1+r') = \frac{1+r'}{1+r^{\text{ss}}}$$

from where we conclude, by the same argument used to prove that capital is increasing, that consumption is increasing during the transition. It also follows from this expression and the fact that the interest rate falls. that the growth rate of consumption is decreasing during the transition. Then the last step is to prove that the growth rate of capital is decreasing during the transition. In order to do this we adapt the proof in Barro and Sala-i-Martin (2004) to discrete time and decreasing returns to scale. The growth rate of $\hat{k}$ is $g_{\hat{k}} = \bar{g}_{\hat{k}}/\gamma$, where $\bar{g}_{\hat{k}} = \hat{y}/\hat{k} - \hat{c}/\hat{k} + (1+\gamma-\delta) = (1-\omega)\hat{y}/\hat{k} - \hat{c}^c/\hat{k} - (1+\gamma-\delta)$. Thus it is enough to show that $\Delta(\bar{g}_{\hat{k}}) < 0$. Differentiating the expression above we get

$$\Delta(\bar{g}_{\hat{k}}) = g_{\hat{k}}(1-\omega)\left[\frac{\partial\hat{y}}{\partial\hat{k}} - \frac{\hat{y}}{\hat{k}}\right] - \frac{\hat{c}^c}{\gamma\hat{k}}\left(g_{\hat{k}} - \gamma g_{\hat{c}^c}\right) = -g_{\hat{k}}\frac{(1-\omega)(1-\alpha)\hat{y}}{\hat{k}} + \frac{\hat{c}^c}{\gamma\hat{k}}\left[(1+r-\gamma) + \frac{\hat{w}-\hat{c}^c}{\gamma\hat{k}} - \gamma g_{\hat{c}^c}\right]$$

Define $A = (1+r) - \gamma(1+g_{\hat{c}^c})$. The only case when $\Delta(\bar{g}_{\hat{k}})$ can be positive is when the term inside the square brackets is positive. Then the rest of the proof assumes that this is the case, and so $\hat{c}^c/(\gamma\hat{k}) < \hat{w}/(\gamma\hat{k}) + A$. But given this inequality, and knowing that $(1-\omega)(1-\alpha) > \upsilon$ and $\gamma > 1$, we get

$$\Delta(\bar{g}_{\hat{k}}) < -\frac{\hat{w}}{\hat{k}}(1+r-\gamma-A) + A\left[A + \frac{\hat{w}-\hat{c}^c}{\hat{k}}\right]$$

Notice that the term inside the first parenthesis is $\gamma g_{\hat{c}^c} > 0$, so, given the inequality above, only if $A > 0$ the RHS can be positive. Then assume $A > 0$ for the rest of the proof. Suppose for now the following is true,

$$\frac{\hat{c}^c}{\hat{k}} > \left[1 + \frac{\hat{w}}{\hat{k}(1+r-\gamma)}\right]A \tag{13}$$

then,

$$\Delta(\bar{g}_{\hat{k}}) < -\frac{\hat{w}}{\hat{k}}(1+r-\gamma-A) + A^2 + \frac{A\hat{w}}{\hat{k}} - \left[1 + \frac{\hat{w}}{\hat{k}(1+r-\gamma)}\right]A^2 < -\frac{\hat{w}}{\hat{k}(1+r-\gamma)}(\gamma g_{\hat{c}^c})^2 < 0$$

so capital grows at a decreasing rate during the transition. Then the last step is to show that expression (13) holds. First, adding the consumers' budget constraints and using the FOC for any period τ we get[32],

$$\hat{c}^c_\tau = \frac{(1+r_\tau)^{1/\sigma}\sum_{t=\tau}^{\infty}\frac{r_t\hat{k}_\tau + \gamma^{(t-\tau)}\hat{w}_t}{\prod_{s=\tau}^{t}(1+r_s)}}{\sum_{t=\tau}^{\infty}\beta^{1/\sigma(t-\tau)}\prod_{s=\tau}^{t}(1+r_s)^{(1/\sigma-1)}}$$

[32]This expression is actually an inequality, because when initial capital is included in the denominator, the RHS falls. Since we are trying to get that inequality this does not affect the proof.

It is easy to see that $\hat{c}^c_\tau$ is decreasing on r_{τ^*}, for any $\tau^* > \tau$, if $\sigma \leq 1$. If $\sigma > 1$ the denominator decreases with r_{τ^*} so it is not clear if in that case $\hat{c}^c_\tau$ is increasing or decreasing on r_{τ^*}. To see that it is decreasing take the case when $\sigma \to \infty$, which is the case when the effect of r_{τ^*} on the denominator is the largest. In that case we have

$$\hat{c}^c_\tau(\sigma \to \infty) = \frac{\sum_{t=\tau}^{\infty} \frac{r_t \hat{k}_\tau + \gamma^{(t-\tau)} \hat{w}_t}{\prod_{s=\tau}^{t}(1+r_s)}}{\sum_{t=\tau}^{\infty} \frac{1}{\prod_{s=\tau}^{t}(1+r_s)}}$$

Notice that $\Delta(r_t \hat{k}_\tau + \gamma^{(t-\tau)} \hat{w}_t) = \Delta r_t(\hat{k}_\tau - \gamma^{t-\tau} \hat{k}_t) + \gamma^{t-\tau}(\gamma - 1)\hat{w}_t > 0$. Thus $\hat{c}^c_\tau$ is decreasing on r_{τ^*} for $\sigma > 1$ as well. As it is increasing on $\hat{w}$, we can replace r_t and $\hat{w}_t$ by their initial levels, r_τ and $\hat{w}_\tau$, and get the following inequality,

$$\hat{c}^c_\tau > \frac{\sum_{t=\tau}^{\infty} \left(\frac{r_\tau \hat{k}_\tau + \gamma^{(t-\tau)} \hat{w}_\tau}{(1+r_\tau)^{t-\tau}} \right)}{\sum_{t=\tau}^{\infty} \left(\beta^{1/\sigma} (1+r_\tau)^{1/\sigma - 1} \right)^{(t-\tau)}} = \left[\hat{k}_\tau + \frac{\hat{w}_\tau}{(1+r_\tau - \gamma)} \right] \left(1 + r_\tau - (\beta(1+r_\tau))^{1/\sigma} \right) = \left[\hat{k}_\tau + \frac{\hat{w}_\tau}{(1+r_\tau - \gamma)} \right] A$$

where the last equality follows from the consumer's FOC. This proves that (13) holds and that the growth rate of capital is decreasing during the transition.

Proof of Proposition 2

The last part of the proposition follows directly from the text, and together with the FOC for labor and the market clearing condition, they constitute a system of 3 equations in 3 unknowns $(\hat{K}, \hat{w}, \hat{C})$. There exists only one solution, and therefore there exists only one steady-state equilibrium. The value of the rest of the endogenous variables $(\hat{y}, \hat{c}^c, \hat{c}^e)$ follows from (1) and the entrepreneur's budget constraint. To show that there is a locally unique path leading to the steady-state we can approximate the dynamic behavior of the nonlinear system by the behavior of the linearized system around the steady state. Using the consumer's Euler equation and the market clearing condition, the linear approximation around the steady state is the following,

$$\begin{aligned} \begin{bmatrix} \Delta \hat{C} \\ \Delta \hat{K} \end{bmatrix} &= \begin{bmatrix} ((\beta(1+r'))^{1/\sigma}/\gamma - 1)\hat{C} & (\beta(1+r'))^{1/\sigma - 1}(\partial r / \partial \hat{K})\hat{C}/(\sigma\gamma) \\ -1 & \alpha \hat{z}^{1-\alpha} \hat{k}^{\alpha - 1} - (1 + \delta - \gamma) \end{bmatrix}_{SS} \begin{bmatrix} \hat{C} - \hat{C}^{\text{ss}} \\ \hat{K} - \hat{K}^{\text{ss}} \end{bmatrix} \\ &= \begin{bmatrix} 0 & -A \\ -1 & B \end{bmatrix} \begin{bmatrix} \hat{C} - \hat{C}^{\text{ss}} \\ \hat{K} - \hat{K}^{\text{ss}} \end{bmatrix} \end{aligned}$$

where, using the expression from the proposition, $B = (1/\Omega)(\gamma^\sigma/\beta - 1 + \delta) - (1 + \delta - \gamma) \geq (\gamma^\sigma/\beta - 1) + (\gamma - 1) > 0$. When the constraint is not binding, $-A = \beta\alpha(\alpha - 1)\hat{z}^{1-\alpha}\hat{K}^{\text{ss}\alpha - 2}\hat{C}^{\text{ss}}/(\sigma\gamma^\sigma) < 0$. When it is binding the IC constraint needs to be used to compute the partial derivative. Doing this and then using the expression from the proposition to evaluate it in steady state we get $-A = -\beta\Omega\alpha\hat{z}^{1-\alpha}\hat{K}^{\text{ss}\alpha-2}\hat{C}^{\text{ss}}/(\sigma\gamma^\sigma) < 0$. Then in either case $A > 0$. After some algebra we get,

$$\hat{K}'' - (2 + B)\hat{K}' + (1 + B - A)\hat{K} + A\hat{K}^{\text{ss}} = 0$$

The roots of the characteristic equation are given by,

$$\lambda_i = \frac{2 + B \pm \sqrt{(2+B)^2 - 4(1 + B - A)}}{2}$$

Since $(2 + B)^2 > 4(1 + B - A)$, the two roots are real. It follows also that the larger root is larger than one, while the smaller is positive and lower than one. This means there is a locally unique path leading to the steady-state.

Proof of Proposition 3

Lemma 1. *Suppose $IC(\hat{K}') = 0$, then $\hat{K}' = \mathrm{K}(\hat{K}) < \mathrm{K}_{\rho=0}(\hat{K})$, where $\mathrm{K}_{\rho=0}(\hat{K})$ is the law of motion for capital assuming $\rho = 0$ in the next period only.*

Proof. Notice from Proposition 2 that $\hat{K}^{\mathrm{ss}} = \mathrm{K}(\hat{K}^{\mathrm{ss}}) \le \mathrm{K}_{\rho=0}(\hat{K}^{\mathrm{ss}})$. Now suppose the statement in the lemma is not true. Then at some point $\hat{K}' = \mathrm{K}(\hat{K}) \ge \mathrm{K}_{\rho=0}(\hat{K})$. As $\hat{K}$ is given, so are $\hat{w}$ and r. But then, from the consumers' budget constraint, $\hat{c}^c \le \hat{c}^c_{\rho=0}$. This, and the fact that $r' < r'_{\rho=0}$ implies $\hat{c}'^c < \hat{c}'^c_{\rho=0}$ in the consumer's Euler equation. Also $\hat{w}' > \hat{w}'_{\rho=0}$, and then, using the budget constraint, $\hat{K}'' = \mathrm{K}(\hat{K}') > \mathrm{K}_{\rho=0}(\hat{K}')$. Repeating the same logic we get that $\hat{K}^{\mathrm{ss}} = \mathrm{K}(\hat{K}^{\mathrm{ss}}) > \mathrm{K}_{\rho=0}(\hat{K}^{\mathrm{ss}})$, a contradiction.. $\square$

For the case when $\Omega = 1$ it is enough to show that $IC^{\mathrm{PE}}(\hat{K})$ is strictly decreasing on $\hat{K}$. To show this it is enough to show that $\tilde{V}(\hat{K}, \hat{K}')$ is strictly decreasing, and $\tilde{V}^d(\hat{K}, \hat{K}')$ is strictly increasing, on $\hat{K}$. First define the sequences $\{g_t\}_{t=0}^{\infty}$ and $\{V_t\}_{t=0}^{\infty}$, where $\forall t$,

$$V_t = \sum_{s=t}^{\infty} \lambda^{s-t} \left[\prod_{h=t}^{s} g_h \right]$$

with $\lambda < 1$. Notice first that $V_{t+1} - V_t = V_{t+1}(1 - \lambda g_t) - g_t$. Now if $\{g_t\}_{t=0}^{\infty}$ is strictly increasing then

$$V_{t+1} > \sum_{s=t+1}^{\infty} \lambda^{s-t-1} g_{t+1}^{s-t} = \frac{g_t}{1 - \lambda g_t} \tag{14}$$

Therefore $V_{t+1} - V_t > 0$, and so $\{V_t\}_{t=0}^{\infty}$ is also strictly increasing. Alternatively, if $\{g_t\}_{t=0}^{\infty}$ is strictly decreasing then the inequality is the opposite in (14), and $\{V_t\}_{t=0}^{\infty}$ is also strictly decreasing. Define $g_t = 1 + g_{\hat{Y}}$, which is strictly decreasing from Proposition 1, and $\lambda = \beta\gamma$, so $V_t = \frac{\tilde{V}(\hat{K}, \hat{K}')}{\omega}$ is strictly decreasing. Finally define $g_t = (1 + g_{\hat{w}})^{-\upsilon/(1-\upsilon)}$, which is strictly increasing from Proposition 1, and $\lambda = \beta\bar{\gamma}$, so $V_t = \frac{\tilde{V}^d(\hat{K}, \hat{K}')}{(1-\upsilon)}$ is strictly increasing.

When $\Omega < 1$ first we show that if $IC_t(\hat{K}_t) = 0$ then $IC_s(\hat{K}_s) = 0 \ \forall s > t$ Suppose this is not true. Then we have $IC_t(\hat{K}_t) = 0$, $IC_{t+i}(\hat{K}_{t+i}) > 0$, for $i = 1, ..., h-1$, and $IC_{t+h}(\hat{K}_{t+h}) = 0$. The last equality comes from Proposition 2. To save notation let us define $V_t = V(\hat{K}_t)$ and $V_{t+m/t} = V^d(\hat{K}_t, \hat{K}_{t+m})$. The last expression is the entrepreneur's utility at $t + m$, when he defaulted at t. Notice that

$$IC_t(\hat{K}_t) = 0 \ \rightarrow \ \frac{\beta\gamma V_{t+1} - \rho\beta\bar{\gamma} V^d_{t+1/t}}{\rho(r_t + \delta)\hat{K}_t} = 1 \ \rightarrow \ \frac{\beta\gamma V_{t+1} - \rho\beta\bar{\gamma} V^d_{t+1/t}}{\rho\alpha\hat{Y}_t} < 1$$

$$IC_{t+1}(\hat{K}_{t+1}) > 0 \ \rightarrow \ \frac{\beta\gamma V_{t+2} - \rho\beta\bar{\gamma} V^d_{t+2/t+1}}{\rho\alpha\hat{Y}_t} > 1$$

because the constraint binds in the first case, and so $(r_t + \delta)\hat{K}_t < \alpha\hat{Y}_t$, and because it does not bind in the second case. Using the fact that $\alpha\hat{Y}_{t+i} = r_{t+i}\hat{K}_{t+i}$ for $i = 1, ...h-1$, these expressions become,

$$\frac{1}{\rho\alpha} \Bigg[\beta\gamma\omega \frac{\hat{Y}_{t+1}}{\hat{Y}_t} \ - \ \rho\beta\bar{\gamma}(1-\upsilon) \left(\frac{\hat{w}_{t+1}}{\hat{w}_t} \right)^{\frac{\upsilon}{\upsilon-1}} + (\beta\gamma)^2 \omega \frac{\hat{Y}_{t+2}}{\hat{Y}_t} - \rho(\beta\bar{\gamma})^2(1-\upsilon) \left(\frac{\hat{w}_{t+2}}{\hat{w}_t} \right)^{\frac{\upsilon}{\upsilon-1}} +$$
$$... + (\beta\gamma)^{h-2} \frac{V_{t+h-1}}{\hat{Y}_t} - \rho(\beta\bar{\gamma})^{h-2} \frac{V_{t+h-1/t}}{\hat{Y}_t} \Bigg] < 1$$

$$\frac{1}{\rho\alpha} \Bigg[\beta\gamma\omega \frac{\hat{Y}_{t+2}}{\hat{Y}_{t+1}} \ - \ \rho\beta\bar{\gamma}(1-\upsilon) \left(\frac{\hat{w}_{t+2}}{\hat{w}_{t+1}} \right)^{\frac{\upsilon}{\upsilon-1}} + (\beta\gamma)^2 \omega \frac{\hat{Y}_{t+3}}{\hat{Y}_{t+1}} - \rho(\beta\bar{\gamma})^2(1-\upsilon) \left(\frac{\hat{w}_{t+3}}{\hat{w}_{t+1}} \right)^{\frac{\upsilon}{\upsilon-1}} +$$
$$... + (\beta\gamma)^{h-2} \frac{V_{t+h}}{\hat{Y}_{t+1}} - \rho(\beta\bar{\gamma})^{h-2} \frac{V_{t+h/t+1}}{\hat{Y}_{t+1}} \Bigg] > 1$$

Since the constraint is not binding we can use the argument used for the case when $\Omega = 1$ to show that each of the first $h-1$ terms in the first expression is greater than the corresponding term in the second expression. Then,

$$\frac{(\beta\gamma)^{h-2}V_{t+h-1} - \rho(\beta\bar{\gamma})^{h-2}V_{t+h-1/t}}{(\beta\gamma)^{h-2}V_{t+h} - \rho(\beta\bar{\gamma})^{h-2}V_{t+h/t+1}} < \frac{\hat{Y}_t}{\hat{Y}_{t+1}} < \frac{\hat{Y}_{t+h-1}}{\hat{Y}_{t+h}}$$

where the last inequality follows from Proposition 1 and Lemma 1. Then we have,

$$m_1 = \frac{(\beta\gamma)^{h-2}V_{t+h-1} - \rho(\beta\bar{\gamma})^{h-2}V_{t+h-1/t}}{\hat{Y}_{t+h-1}} < \frac{(\beta\gamma)^{h-2}V_{t+h} - \rho(\beta\bar{\gamma})^{h-2}V_{t+h/t+1}}{\hat{Y}_{t+h}} = m_2$$

Now define the following expressions,

$$A_1 = \rho(\beta\bar{\gamma})^{h-1}\left[\frac{V^d_{t+h/t+h-1} - V^d_{t+h/t}}{\hat{Y}_{t+h-1}}\right] > 0$$

$$A_2 = \rho(\beta\bar{\gamma})^{h-1}\left[\frac{V^d_{t+h+1/t+h} - V^d_{t+h+1/t+1}}{\hat{Y}_{t+h}}\right] > 0$$

Suppose for now that $A_1 > A_2$, so $m_1 - A_1 < m_2 - A_2$. Since $\gamma > \bar{\gamma}$,

$$m_1 - A_1 > (\beta\gamma)^{h-2}\omega - \frac{\rho(\beta\bar{\gamma})^{h-2}\hat{z}\hat{K}_t^{\alpha}n^{\upsilon}_{t+h-1}}{\hat{Y}_{t+h-1}} + \frac{(\beta\gamma)^{h-2}\left(\beta\gamma V_{t+h} - \rho\beta\bar{\gamma}V^d_{t+h/t+h-1}\right)}{\hat{Y}_{t+h-1}}$$

For m_2 we have,

$$m_2 - A_2 = \frac{(\beta\gamma)^{h-2}\left((1-\upsilon)\hat{Y}_{t+h} - (r_{t+h}+\delta)\hat{K}_{t+h}\right) - \rho(\beta\bar{\gamma})^{h-2}\hat{z}\hat{K}^{\alpha}_{t+1}n^{\upsilon}_{t+h} + (\beta\gamma)^{h-1}V_{t+h+1} - \rho(\beta\bar{\gamma})^{h-1}V^d_{t+h+1/t+h}}{\hat{Y}_{t+h}}$$

Using again $\gamma > \bar{\gamma}$, and also that $\alpha\hat{Y}_{t+h} = (r_{t+h}+\delta)\hat{K}_{t+h} = (\beta\gamma V_{t+h+1} - \rho\beta\bar{\gamma}V^d_{t+h+1/t+h})/\rho$ because $t+h$ is the first period when $IC(\hat{K}) = 0$,

$$m_2 - A_2 > (\beta\gamma)^{h-2}(1-\upsilon) - \rho\alpha(\beta\gamma)^{h-2}\left(\frac{1}{\rho} - 1\right) - \frac{\rho(\beta\bar{\gamma})^{h-2}\hat{z}\hat{K}^{\alpha}_{t+1}n^{\upsilon}_{t+h}}{\hat{Y}_{t+1}}$$

Notice that in the last expressions n is different from one, and depends on current wages and the stock of capital the entrepreneur kept when defaulted. Because $m_1 - A_1 < m_2 - A_2$, we have

$$\frac{\left(\beta\gamma V_{t+h} - \rho\beta\gamma V^d_{t+h/t+h-1}\right)}{\alpha\rho\hat{Y}_{t+h-1}} < 1 + \frac{1}{\alpha}\left(\frac{\bar{\gamma}}{\gamma}\right)^{h-2}\left(\frac{\hat{z}\hat{K}_t^{\alpha}n^{\upsilon}_{t+h-1}}{\hat{Y}_{t+h-1}} - \frac{\hat{z}\hat{K}^{\alpha}_{t+1}n^{\upsilon}_{t+h}}{\hat{Y}_{t+h}}\right) < 1$$

where the last inequality follows from the fact that Proposition 1 and Lemma 1 imply, when using the optimal demand for labor,

$$\frac{\hat{z}\hat{K}_t^{\alpha}n^{\upsilon}_{t+h-1}}{\hat{Y}_{t+h-1}} - \frac{\hat{z}\hat{K}^{\alpha}_{t+1}n^{\upsilon}_{t+h}}{\hat{Y}_{t+h}} = \left(\frac{\hat{Y}_t}{\hat{Y}_{t+h-1}}\right)^{\frac{1}{1-\upsilon}} - \left(\frac{\hat{Y}_{t+1}}{\hat{Y}_{t+h}}\right)^{\frac{1}{1-\upsilon}} < 0.$$

But this contradicts $IC_{t+h-1}(\hat{K}_{t+h-1}) > 0$, so if $IC_t(\hat{K}_t) = 0$ then $IC_s(\hat{K}_s) = 0\ \forall s > t$. Therefore to get this contradiction we need to show $A_1 > A_2$, or $(A_1 - A_2)/(\beta\bar{\gamma})^{h-1} > 0$. Using the optimal demand for labor we have,

$$\begin{aligned}
\frac{A_1-A_2}{(\beta\bar{\gamma})^{h-1}} &= \left(\frac{\hat{Y}_{t+h-1}^{\frac{v}{1-v}}-\hat{Y}_{t}^{\frac{v}{1-v}}}{\hat{Y}_{t+h-1}}\right)\left(\hat{Y}_{t+h}^{\frac{-1}{1-v}}+\beta\bar{\gamma}\sum_{s=0}^{\infty}(\beta\bar{\gamma})^s\hat{Y}_{t+h+1+s}^{\frac{-1}{1-v}}\right)-\left(\frac{\hat{Y}_{t+h}^{\frac{v}{1-v}}-\hat{Y}_{t+1}^{\frac{v}{1-v}}}{\hat{Y}_{t+h}}\right)\sum_{s=0}^{\infty}(\beta\bar{\gamma})^s\hat{Y}_{t+h+1+s}^{\frac{-1}{1-v}} \\
&> \left(\frac{\hat{Y}_{t+h}^{\frac{v}{1-v}}-\hat{Y}_{t+1}^{\frac{v}{1-v}}}{\hat{Y}_{t+h}}\right)\left(\hat{Y}_{t+h}^{\frac{-1}{1-v}}-(1-\beta\bar{\gamma})\sum_{s=0}^{\infty}(\beta\bar{\gamma})^s\hat{Y}_{t+h+1+s}^{\frac{-1}{1-v}}\right) \\
&> \left(\frac{\hat{Y}_{t+h}^{\frac{v}{1-v}}-\hat{Y}_{t+1}^{\frac{v}{1-v}}}{\hat{Y}_{t+h}}\right)\left(\hat{Y}_{t+h}^{\frac{-1}{1-v}}-\hat{Y}_{t+h+1}^{\frac{-1}{1-v}}\right)>0
\end{aligned}$$

where the first inequality follows from Proposition 1 and Lemma 1.

Now it is left to show $\hat{K}^*>0$. Take the limit of $IC(\hat{K})$ when $\hat{K}$ goes to zero. The only term that does not converge to zero, independently of ρ, is $V(\hat{K},\hat{K}')$ because of Proposition 2. It follows that $\forall\rho\in[0,1]$, $lim_{\hat{K}\to 0}IC(\hat{K})>0$. Then it is not the case that the constraint is always binding, implying the existence of $\hat{K}^*$ and the statement in the proposition.

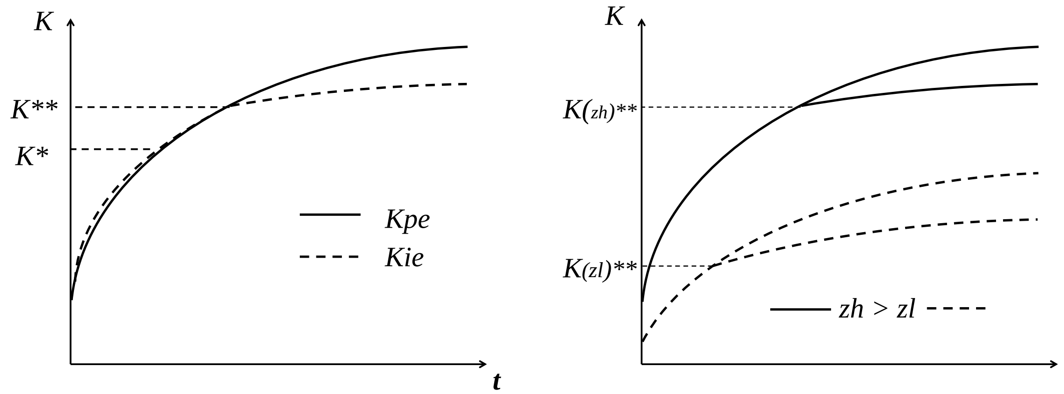
Proof of Proposition 4

Suppose $\hat{K}_t>\hat{\mathbf{K}}_t$ if $\hat{K}_t\leq K^*$. Then, by Proposition 2 and Lemma 1, we know $\exists!s$ where $\hat{K}_s=\hat{\mathbf{K}}_s$ and $\hat{K}_s>K^*$. Call this level $\hat{K}^{**}$. Thus it is enough to show that $\hat{K}_t>\hat{\mathbf{K}}_t$ if $\hat{K}_t\leq K^*$. Since the constraint is not binding when $\hat{K}_t\leq K^*$, remember that $(r_t+\delta)=\alpha\hat{Y}_t/\hat{K}_t$. Suppose $\hat{C}_0^c\geq\hat{\mathbf{C}}_0^c$, since $\hat{K}_0=\hat{\mathbf{K}}_0$, from the market clearing condition we have $\hat{K}_1\leq\hat{\mathbf{K}}_1$. But then $r_1\geq\mathbf{r}_1$, implying using the consumers' Euler equation, $\hat{C}_1^c\geq\hat{\mathbf{C}}_1^c$ (and $\hat{C}_1^c/\hat{C}_0^c\geq\hat{\mathbf{C}}_1^c/\hat{\mathbf{C}}_0^c$). Repeating this argument we obtain $\hat{K}^*\leq\hat{\mathbf{K}}^*$ and $\hat{C}^{*c}/\hat{C}_{-1}^{*c}\geq\hat{\mathbf{C}}^{*c}/\hat{\mathbf{C}}_{-1}^{*c}$. But we need $r^*<\mathbf{r}^*$ to have $IC(\hat{K}^*)=0$, implying, using the consumers' Euler equation that $\hat{C}^{*c}/\hat{C}_{-1}^{*c}<\hat{\mathbf{C}}^{*c}/\hat{\mathbf{C}}_{-1}^{*c}$, which is a contradiction. The only possibility is then $\hat{C}_0^c<\hat{\mathbf{C}}_0^c$, when, using the same argument above, $\hat{K}_t>\hat{\mathbf{K}}_t$ if $\hat{K}_t\leq K^*$.

Appendix C: Data

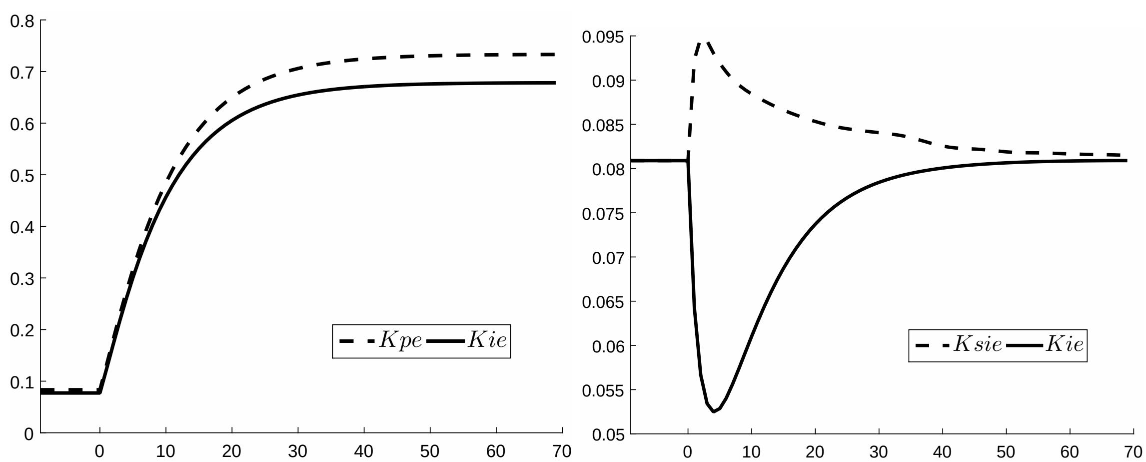
		Legal Formalism	Contract Enf. (Ln)	Executive Constraints	English LO	Pop. Density in 1500 (Ln)	GDP pc 2006 (Ln)	GDP pc 1950 (Ln)	Sec. Enroll. 1960 (Ln)
1	Algeria	..	3.87	2.68	0	1.95	8.13	7.22	2.08
2	Argentina	5.40	3.58	5.70	0	-2.21	9.18	8.51	3.14
3	Australia	1.80	4.48	7.00	1	-3.51	10.10	8.91	3.93
4	Bangladesh	3.24	..	4.37	1	3.17	6.95	6.29	2.08
5	Bolivia	5.75	..	7.00	0	-0.19	7.93	7.56	2.48
6	Botswana	4.08	4.24	6.47	1	-1.97	8.44	5.85	0.00
7	Brazil	3.06	2.60	5.90	0	-2.12	8.71	7.42	2.40
8	Canada	2.09	4.53	7.00	1	-3.91	10.13	8.89	3.83
9	Chile	4.57	3.71	6.15	0	-0.22	9.43	8.21	3.18
10	Colombia	4.11	4.17	6.20	0	-0.04	8.69	7.67	2.48
11	Costa Rica	5.48	3.22	7.00	0	0.43	8.92	7.58	3.04
12	Cte d'Ivoire	3.65	..	2.57	0	1.44	7.00	6.95	..
13	Dominican Republic	4.08	2.56	5.50	0	0.38	8.30	6.93	1.95
14	Ecuador	4.92	2.97	6.60	0	0.77	8.23	7.53	2.48
15	Egypt	3.79	3.35	3.00	0	4.61	8.12	6.81	2.77
16	El Salvador	4.60	3.63	5.00	0	0.43	7.95	7.31	2.56
17	Ghana	2.65	..	3.67	1	1.44	7.32	7.02	1.61
18	Guatemala	5.68	3.60	4.50	0	0.43	8.35	7.64	1.95
19	Honduras	4.90	3.61	5.00	0	0.43	7.69	7.18	2.08
20	India	3.34	..	7.00	1	3.17	7.87	6.43	3.00
21	Indonesia	3.90	3.22	3.53	0	1.45	8.30	6.69	1.79
22	Jamaica	2.34	4.23	7.00	1	1.53	8.25	7.19	3.81
23	Kenya	3.09	..	3.63	1	0.97	6.97	6.48	0.69
24	Malawi	2.95	..	3.53	1	-0.24	6.48	5.78	0.00
25	Malaysia	2.34	3.88	4.42	1	0.20	9.17	7.35	2.94
26	Mexico	4.71	4.28	4.70	0	0.96	8.96	7.77	2.40
27	Morocco	4.71	3.74	2.74	0	2.21	8.10	7.28	1.61
28	Mozambique	4.49	..	3.63	0	0.25	7.58	7.03	0.69
29	Namibia	3.82	3.81	5.00	1	-1.97	8.36	7.68	..
30	New Zealand	1.58	4.51	7.00	1	-0.99	9.82	9.04	4.29
31	Nigeria	3.19	..	2.56	1	1.44	7.24	6.62	1.39
32	Pakistan	3.76	..	4.68	1	3.17	7.68	6.47	2.40
33	Panama	5.84	3.76	5.40	0	0.43	8.72	7.56	3.37
34	Paraguay	5.91	2.53	5.45	0	-0.69	8.03	7.37	2.40
35	Peru	5.60	3.73	5.05	0	0.44	8.44	7.74	2.71
36	Philippines	5.00	2.86	6.26	1	0.52	7.92	6.98	3.26
37	Senegal	4.72	..	3.95	0	1.44	7.27	7.14	1.10
38	Singapore	2.50	4.57	3.00	1	-2.41	10.18	7.70	3.47
39	South Africa	1.68	3.68	7.00	1	-0.71	8.41	7.84	2.71
40	Sri Lanka	3.78	3.82	5.11	1	2.74	8.39	7.13	3.30
41	Swaziland	3.30	..	1.37	1	-0.71	7.99	6.58	1.61
42	Tanzania	3.82	..	3.00	1	0.68	6.51	6.05	0.69
43	Tunisia	4.05	4.04	2.79	0	2.46	8.63	7.02	2.48
44	Trinidad and Tobago	2.98	..	7.00	1	0.38	9.90	8.21	3.18
45	Uganda	2.61	..	2.37	1	2.02	6.82	6.53	1.10
46	United States	2.62	4.45	7.00	1	-2.41	10.34	9.17	4.45
47	Uruguay	4.05	3.35	7.00	0	-2.21	9.05	8.45	3.61
48	Venezuela	6.01	2.57	5.65	0	-0.82	9.17	8.92	3.04
49	Vietnam	3.25	..	3.00	0	1.81	7.87	6.49	..
50	Zambia	2.13	..	4.16	1	-0.24	6.65	6.49	0.69
51	Zimbabwe	3.11	..	2.74	1	-0.24	6.87	6.55	1.79

Figures and Tables



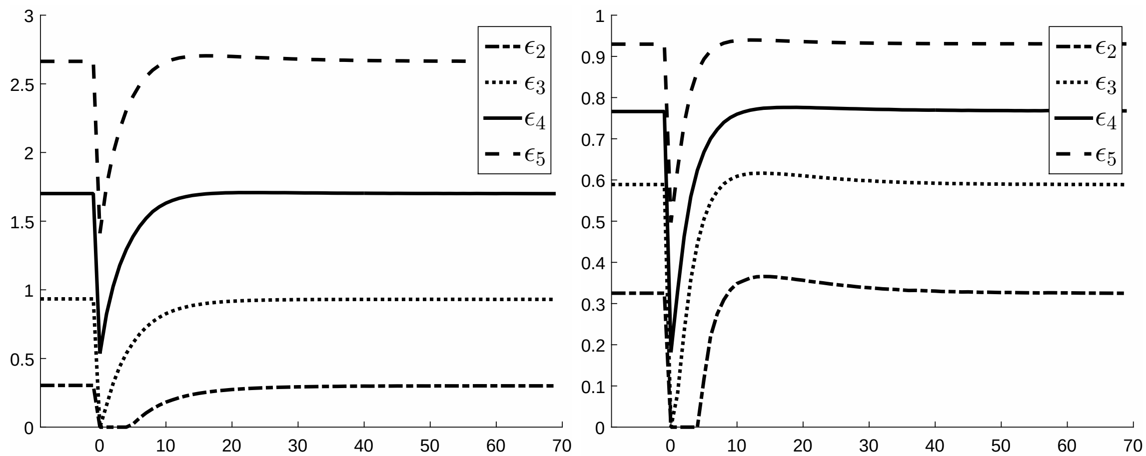
Notes: theoretical model predictions for the transition to the steady-state. Left: path for capital per effective unit of labor under perfect enforcement (solid line) and under imperfect enforcement (dashed line). Right: paths for capital per effective unit of labor for high (solid lines) and low (dashed lines) levels of z.

Figure 1: Imperfect vs. Perfect Enforceability Equilibriums in the Theoretical Model.



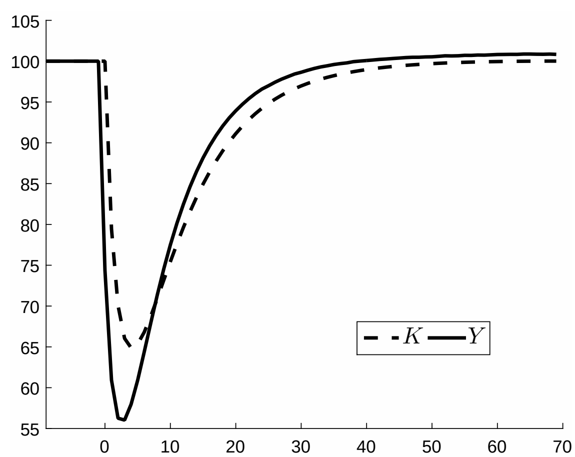
Notes: quantitative model results for the transition between steady-states. Left: path for capital under perfect enforcement (dashed line) and imperfect enforcement (solid line). Right: percentage difference between capital under perfect and imperfect enforcement (solid line), and under perfect enforcement and imperfect enforcement when the static constraint (6) replaces the dynamic constraint (5) (dashed line).

Figure 2: Imperfect vs. Perfect Enforceability Equilibriums in the Quantitative Model.



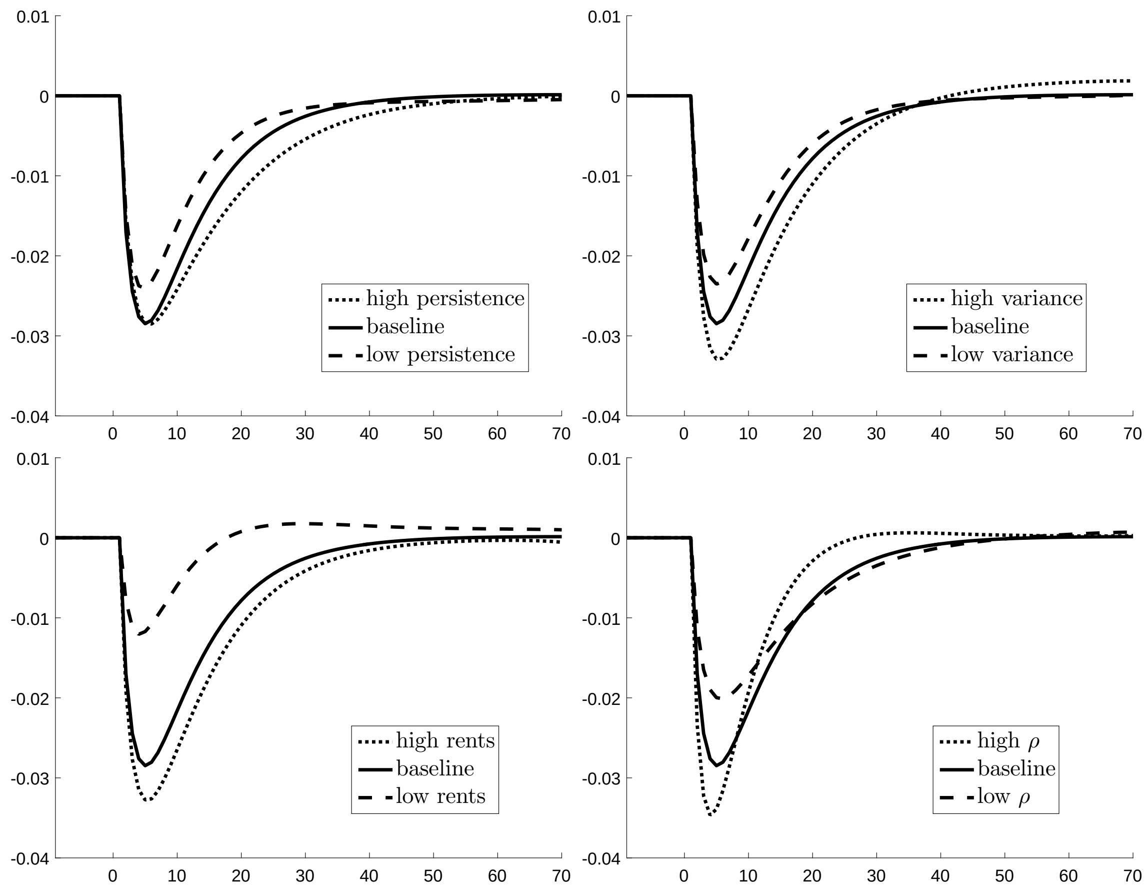
Notes: the figure shows how endogenous financial constraints affect agents with different productivity levels in the quantitative model. Left: minimum collateral, as a fraction of aggregate capital, necessary to finance the efficient level of capital with external resources for individual entrepreneurs with different levels of productivity: from the second lowest (ϵ_2) to the highest (ϵ_5). The lowest level is not shown because the constraint never binds in that case. Right: fraction of entrepreneurs that are not able to finance the efficient level of capital for each productivity level.

Figure 3: Binding Pattern of Endogenous Financial Constraints in the Quantitative Model.



Notes: differences in aggregate capital (dashed line) and output (solid line) between the perfect and imperfect enforcement equilibriums in the transition between steady-states in the quantitative model, as a percentage of steady-state differences.

Figure 4: Misallocation of Capital in the Quantitative Model.



Notes: percentage difference between capital under perfect and imperfect enforcement for different parameter values in the transition between steady-states. Steady-state differences are normalized to zero, as they vary with the value of the parameters. High (low) persistence means an AR(1) coefficient for productivity shocks of 0.95 (0.75) instead of 0.85 used in the baseline calibration. High (low) variance means a standard deviation for productivity shocks that is 10% higher (lower) than in the baseline. High (low) rents means a value for the span of control of 0.24 (0.18) instead of 0.21 used in the baseline calibration. High (low) ρ means a value for this parameter of 0.85 (0.65) instead of 0.75 used in the baseline calibration.

Figure 5: Sensitivity Analysis, Quantitative Model.

	$\log \hat{Y}^{ss}$	$\log \hat{Y}_t$			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
log(1-ρ)	1.00***	0.49***	0.33***	−0.07***	0.33***
	0.00	*0.01*	*0.00*	*0.01*	*0.00*
log $\hat{z}$	1.00***	1.01***	0.55***	0.55***	0.59***
	0.00	*0.02*	*0.00*	*0.00*	*0.02*
log $\hat{Y}_{t-T}$			0.46***	0.30***	0.47***
			0.00	*0.00*	*0.01*
log $\hat{Y}_{t-T}$ $\times$ **log(1-ρ)**				0.80***	
				0.02	
log $\hat{Y}_{t-T}$ $\times$ **log** $\hat{z}$					−0.07**
					0.03
R^2	0.99	0.49	0.97	0.98	0.97
Observations	5000	5000	5000	5000	5000

Notes: regression results from estimating (7), (8), and (9), using data generated by numerically solving the deterministic model presented in section 2. Standard errors are in italics, ***,**, and * indicate significance at 1%, 5%, and 10% levels.

Table 1: Regression Results using Simulated Data.

	Legal Formalism	Contract Enforcement	Executive Constraints	English LO	Pop. Density 1500 (Ln)	GDP pc. 2006 (Ln)	GDP pc. 1950 (Ln)	Sec. Enroll. 1960 (Ln)
Average	3.81	3.67	4.84	0.49	0.31	8.26	7.32	2.38
Median	3.81	3.73	5	0	0.43	8.25	7.22	2.48
Max	6.01	4.57	7	1	4.61	10.34	9.17	4.45
Min	1.58	2.53	1	0	-3.91	6.48	5.78	0
Countries	50	33	51	51	51	51	51	48
					Correlation Matrix			
Legal Formalism		-0.56	-0.23	-0.74	0.31	-0.51	-0.24	-0.40
Contract Enforcement			0.05	0.55	-0.23	0.55	0.27	0.31
Executive Constraints				0.34	-0.57	0.29	0.45	0.38
English LO					-0.35	0.43	0.15	0.38
Pop. Density 1500 (Ln)						-0.67	-0.59	-0.29
GDP pc. 2006 (Ln)							0.76	0.63
GDP pc. 1950 (Ln)								0.75

Notes: main statistics of the variables used in the empirical section. In Appendix C we show the values of each variable for every country in the sample.

Table 2: Descriptive Statistics.

	Contract Enforcement		*Legal Formalism (-)*		*Executive Constraints*	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
English Legal Origin	0.65^{***}	0.68^{***}	1.72^{***}	1.78^{***}	-0.12	0.33
	0.22	*0.21*	*0.24*	*0.26*	*0.43*	*0.37*
Log Population Density in 1500	-0.00	0.05	-0.04	0.00	-0.41^{***}	-0.10
	0.05	*0.06*	*0.07*	*0.09*	*0.11*	*0.18*
Log GDP per capita in 1950		0.21		0.15		1.08^{***}
		0.18		*0.20*		*0.33*
R^2	0.28	0.33	0.53	0.54	0.19	0.36
Observations	33	33	50	50	51	51

Notes: first-stage regressions. Dependent variables are contract enforcement, legal formalism, and executive constraints. Robust standard errors are in italics, ***,**, and * indicate significance at 1%, 5%, and 10% levels.

Table 3: First-Stage Regressions.

	TSLS 1-ρ: *Contract Enforcement* z: *Executive Constraints*			*TSLS* 1-ρ: *Legal Formalism (-)* z: *Executive Constraints*			*OLS* 1-ρ: *English Origin* z: *Pop. Density in 1500 (-)*		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
1-ρ	0.04	0.29	0.22	-0.08	0.02	-0.09	-0.29	-0.09	-0.13
	0.59	*0.29*	*0.31*	*0.18*	*0.12*	*0.15*	*0.23*	*0.14*	*0.14*
z	0.57^{***}	0.34	0.27^{*}	0.81^{***}	0.54	0.40^{**}	0.31^{***}	0.13^{*}	0.14^{*}
	0.22	*0.22*	*0.15*	*0.24*	*0.45*	*0.20*	*0.06*	*0.08*	*0.08*
log Y_{t-T}		$0.37^{\dagger\dagger}$	0.42^{*}		$0.40^{\dagger}$	0.39^{*}		$0.38^{*\dagger\dagger\dagger}$	0.11
		0.30	*0.25*		*0.34*	*0.21*		*0.22*	*0.23*
log Y_{t-T} $\times$ **(1-ρ)**			0.27			0.25^{*}			0.36^{**}
			0.25			*0.15*			*0.14*
Richest country in t-T (‡)									
1-ρ			0.71^{***}			0.36^{**}			0.54^{*}
			0.27			*0.17*			*0.28*
R^2		0.32	0.45		0.32	0.50	0.32	0.78	0.79
Observations	33	32	32	50	47	47	51	48	48

Notes: second-stage results. The dependent variable is log Y_t, which is GDP per capita in 2006. t-T=1950. GDP per capita in 1950 and (1-ρ) are normalized by their averages. (‡): coefficient and significance levels normalizing initial output by the value of the richest country in 1950. Robust standard errors are in italics; ***,**, and * indicate significance at 1%, 5%, and 10% levels. †††,††, and † indicate that the coefficient is significantly different from one at 1%, 5%, and 10% levels. Log of secondary enrollment in 1960 is included as an additional regressor in all specifications except (1), (4), and (7).

Table 4: Income and CI Institutions, Regression Results.